# Project Deliverable

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**Title**

# D3.1: Coalition analysis methods reported in literature

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**Abstract :**

This deliverable is the outcome of the first step of Task 3.1, part of the WP3: “Coalitional Games in Systems of Systems”. The text provides a summary of the fundamental concepts of coalitional game theory and methods found in the relevant literature, in particular those most related with the case studies considered within the Dymasos project and with control engineering applications in general. Drawing from the well established framework of the control of large-scale systems, a brief literature survey traces an ideal path from the optimal model partitioning to the distributed control schemes whose principles happen to be close to the idea of coalitional control.

The deliverable serves the purpose of consolidating the background knowledge leading up to the consequent steps of Task 3.1 and Task 3.2, i.e., the development of tailored methods for the analysis and the synthesis of control strategies for dynamically evolving coalitional structures.

**Keywords :**

Coalitional control, game theory, distributed control, hierarchical control, system partitioning

# Disclaimer



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# Revision History

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# The DYMASOS Project

The well-being of the citizens in Europe depends on the reliable and efficient functioning of large interconnected systems, such as electric power systems, air traffic control, railway systems, large industrial production plants, etc. Such large systems consist of many interacting components. The sub-systems are usually managed locally and independently, according to different policies and priorities. The dynamic interaction of the locally managed components gives rise to complex behaviour and can lead to large-scale disruptions as e.g. black-outs in the electric grid.

Large interconnected systems with autonomously acting sub-units are called systems of systems. DYMASOS addresses systems of systems where the elements of the overall system are coupled by flows of physical quantities, e.g. electric power, steam or hot water, etc.

Within the project, new methods for the distributed management of large physically connected systems with local management and global coordination will be developed.

**The DYMASOS Consortium** consists of:

| Participant no. | Participant organisation name | Participant short name | Country |
|---|---|---|---|
| 1 | Technische Universität Dortmund | TUDO | Germany |
| 2 | BASF SE | BASF | Germany |
| 3 | HEP-Operator distribucijskog sustava d.o.o | HEP | Croatia |
| 4 | INEOS Köln GmbH | INEOS | Germany |
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# Contents



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# 1 Executive summary

This deliverable is the outcome of the first step of Task 3.1 (part of the WP3 *Coalitional Games in Systems of Systems*), namely the inquiry of different coalition analysis methods reported in the open literature and the selection of the appropriate methods for the study cases. The text provides a summary of the fundamental concepts of *coalitional game theory*, in particular those most related with the case studies considered within the Dymasos project and with control engineering applications in general. Drawing from the well established framework of the control of large-scale systems, a brief literature survey traces an ideal path from the optimal model partitioning to the distributed control schemes whose principles happen to be close to the idea of coalitional control.

This document serves the purpose of consolidating the background knowledge leading up to the consequent steps of Task 3.1 and Task 3.2, i.e., the development of tailored methods for the analysis and the synthesis of control strategies for dynamically evolving coalitional structures. Furthermore, in order to reveal possible guidelines for the development of solutions for the Dymasos case studies (especially the HEP and the AYESA ones), the application of game theory to the smart grid is explored in §5. In particular, the open issues evidenced by studies concerning the management of microgrids and storage devices connected to the grid are presented.

# 2 Introduction

Any system considered within feedback control theory has a well-defined global organizational objective. This does not exclude the possibility that certain components of the system have their own interests, or that they have incorrect models of the rest of the system [1]. The introduction of new technologies in data acquisition (e.g., wireless networks, smart sensors) and in database management (e.g., cloud computing) provides a means of sharing measures and other information of a large-scale plant in an efficient and flexible way [2, 3]. This improvement in the computational and communicational capabilities provided to local control devices constitutes an additional impulse towards a (new) distributed (coalitional) approach to large-scale control problems, already motivated by inherent logistical issues and structural constraints. Indeed, in most cases centralized strategies do not exploit the inherent structure of the system, leading to oversized computational and communicational requirements.

For these reasons, a lot of effort has been dedicated to the development of model-based distributed controllers for large-scale systems (see, e.g., [4, 5, 6, 7]). The common objective of these decentralized and distributed approaches is to achieve a global optimal performance comparable to that expected through the use of a centralized controller, with the added scalability and robustness of a distributed implementation [2]. This usually involves a tradeoff between performance loss and a less complex implementation.

The stronger the interaction among different parts of a system, the denser the communication required between the control agents (the extreme case being equivalent to centralized control) [8]. In several cases the variables of a system can be grouped to highlight weakly coupled blocks, often revealing a natural topology. Within each block (*neighborhood*) dynamic interactions propagate quickly, affecting the rest of the system on a longer time scale [9]. Whenever possible it is desirable —for ease of implementation and reduction of the communication overhead— to formulate control laws based exclusively on local information [10, 11]. An interesting challenge is the online identification of subsystems' interactions [8], and the consequent adjustment of the control topology (thus varying its associated computational and communicational requirements). This is the rationale leading to *coalitional* control, where the strategy is adapted to the varying conditions of coupling between the agents, promoting the formation of coalitions among those most concerned [12].

On these premises, the literature survey included in the present deliverable has been organized in order to trace a path from the issues related with one of the fundamental steps in the design of distributed controllers, i.e., model partitioning (§4.1), to those solutions whose principles delineate the idea of coalitional control (§4.2–4.3). The survey concludes with a presentation of the works relating the classic notions of game theory to the control engineering field (§4.4).

In order to consolidate the background knowledge required for the accomplishment of Task 3.1 and Task 3.2, and to contribute to the development of solutions for the Dymasos case studies, the first part of this document consists of a presentation of the core concepts of game theory (§3). These concepts constitute a basis for the analysis of the interaction of control agents, as well as for the design of cooperative mechanisms for the management of complex systems. Some of these notions, in particular those developed within the framework of noncooperative games (e.g., the Nash equilibrium) have been extensively applied in the distributed control literature (§3.2–3.7). Notions related with cooperative games (§3.8–3.14), however, are in general not tailored for dynamical environments. As such, their application in control engineering is seldom encountered. Nevertheless, the cooperative games framework is being extended towards the control engineering world thanks to a growing number of pioneering works, to which the outcome of WP3 is expected to contribute.

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# 3 Coalitional control and game theory

*The formation of coalitions among the control agents in a distributed control framework is the foundation of* coalitional control*. This section presents the notions of game theory most related with coalitional control and, in general, with distributed control. For its wide application in the design and the analysis of distributed control strategies, the concept of Nash equilibrium is first introduced. The remainder of the section is dedicated to the basic tools for the analysis of coalitional stability. Finally, the dynamic coalition formation problem is briefly introduced.*

## 3.1 Introduction

Game theory aims at mathematically characterizing the behavior of independent agents interacting within a complex environment, in the presence of conflicting interests. Two main branches can be identified in this field, addressing *noncooperative* and *cooperative* games, respectively. Noncooperative games model situations in which a number of independent agents, that have (partially) conflicting interests, optimize their individual strategy on the basis of a utility index affected by other agents' actions, *without any coordination* between them. Notice that this does not preclude cooperation: indeed, cooperation may arise from the specific architecture of the game, without any agreement among the agents. A particular class of games is related to incentive system design. Here, the higher level of an organization chooses a reward scheme and the lower levels make decisions trying to maximize their individual reward. The problem consists in *choosing a reward scheme so that the lower level decision makers end up minimizing a global cost function* [1]. A typical example of application is distributed resource allocation [13]. Cooperative games are appropriate to model situations in which some commonalities are found among the agents [14]. As a consequence, the individual benefit can be improved through the joint operation of the players. Especially related with the Dymasos project are the *dynamic coalition formation* games. These games contemplate the evolution of the cooperative structure implemented by the players in order to operate jointly. Such evolution can be ascribed, e.g., to variations in the degree of coupling among different parts of the system, to players that enter or leave the game in a plug & play setting.

In game theory, the term *strategy* designates a well-defined sequence of actions (also *moves*) applied by a given agent, generally resulting from a particular response algorithm dictating the behavior of the agent. Any game provides a given set of actions available for each agent to pick at each turn, which can be translated into the control engineering terminology as the input constraint set. An *action* is intended as an input applied to the system.

A basic distinction is made between actions applied in a sequential or simultaneous fashion. The first category allows to consider the case where the players are aware of the strategy implemented by others. Thus the outcome of *sequential* games (also known as dynamic games, or games in extensive form) can be represented by trees. The second category, known as games in *strategic* (or normal) form, are used to model scenarios in which the players act simultaneously or, in general, are not aware of the actions

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taken by others. The outcome of such games is generally represented on a table [14].

Games can also be classified depending on the quality of information available to the players: a *complete information* game is characterized by the knowledge —by each player— of the strategies and payoffs *available* to the other players; this does not necessarily include information of the strategy actually implemented by the others. The designation *perfect information* identifies the complete knowledge of actions taken by all players [14].[1] To give an example, in a game with complete and imperfect information players are aware of everyone's available actions and corresponding payoffs, but their decisions cannot be based on the knowledge of other players' decisions.

*Static games* model situations in which the agents apply a one-shot strategy, as opposed to *repeated game* models. The scenarios encountered in control engineering show, in general, analogies with both dynamic games —in which time has a fundamental role in the decision making (the strategy can be based on the knowledge of past implemented actions)— and repeated games, since the agents can act more than once. One class of games is dedicated to the analysis of *infinitely repeated* games, where the horizon is not known a priori (in this class of games, the focus is on the existence of a winning strategy) [14].

One of the foundations of game theory is the assumption that the agents are *rational*, i.e., all the actions taken are meant to improve utility. However, due to the suboptimality of the control strategy, or other factors such as failures or delays, this assumption does not always hold in practice. In general, the notion of rationality may be better interpreted as each player's knowledge of its objectives, evaluated on the basis of its own value system to synthesize the strategy to be implemented [14]. It is important to design algorithms capable of preventing the system to deviate from the desired equilibrium due to possible nonrational decisions. Game theory provides proper frameworks to deal with such errors, e.g., the concept of perturbed equilibrium, games with imperfect information or imperfect observability. The study of games with *bounded rationality* is an emerging field that addresses situations where nonrational decisions may be taken [15].

## 3.2 Noncooperative games

A game is defined through three elements: a set $\mathcal{N}$ of agents (players), a set of allowed actions $\mathcal{A}_i$ and a utility function $u_i$ for each agent $i \in \mathcal{N}$. In a noncooperative game, each agent $i$ chooses the action $a_i \in \mathcal{A}_i$ that maximizes its utility $u_i(a_i, \mathbf{a}_{-i})$, which depends as well on the actions $\mathbf{a}_{-i}$ of the rest of the agents. When the game is dynamic, additional elements are also reflected in the utility function, such as time, past actions, information sets [15]. Sequences of actions applied in a deterministic manner are defined as *pure strategies*; sequences of actions picked according to a given probability distribution are defined as *mixed strategies*.

[1] In simultaneous games the players can only be aware of past actions.

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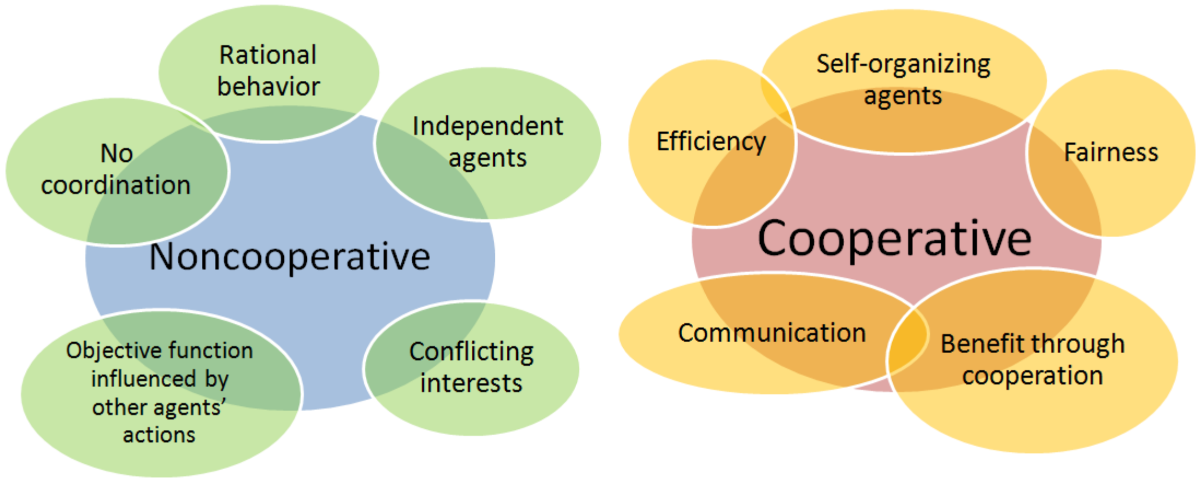


Figure 1: Noncooperative vs. cooperative game theory

## 3.3 Dominant and dominated strategies

From the point of view of each player, strategies may be characterized as dominant and dominated. A strategy is referred to as *dominant* if the player has the incentive to play it regardless of the strategies played by other players. On the other hand, a strategy is *dominated* if the player always has better alternatives regardless of the strategies played by the others.

## 3.4 Equilibria in noncooperative games

The solution of a strategic game (i.e., where the players are not aware of the actions taken by others) is represented by possible equilibria of the strategies applied by the agents. The Nash equilibrium corresponds to a stable state in which no agent can improve its utility by changing its strategy, once the other agents' choices are fixed. It can be reached with little or absent coordination in decentralized settings, and may be not unique. In particular, the Nash equilibrium of a static noncooperative game with pure strategy is a set of actions $\mathbf{a}^* \in \mathcal{A} \equiv \mathcal{A}_1 \times \mathcal{A}_2 \times \ldots \times \mathcal{A}_{|\mathcal{N}|}$ such that for all the agents $i \in \mathcal{N}$ holds

$$u_i(a_i^*, \mathbf{a}_{-i}^*) \geq u_i(a_i, \mathbf{a}_{-i}^*),\ \forall a_i \in \mathcal{A}_i \tag{1}$$

For mixed strategies a similar definition is given, based on the probability distribution of the actions to take. A Nash equilibrium is guaranteed to exist only in mixed strategy games. A Nash equilibrium cannot involve any dominated strategy.

In presence of multiple Nash equilibria, several metrics can be used to study the efficiency of each one of them, such as the price of anarchy and the price of stability [16, 15].

For sequential games, a different concept —known as *subgame perfect equilibrium*— is used. In such games, agents synthesize their strategies based on the actions already taken by others, and on the consequent range of future outcomes. Thus, each agent will focus on the subtree —whose root represents

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the current state— showing the possible evolution of the game.

## 3.5 Bargaining theory

Bargaining theory studies situations in which two or more agents have a common interest to cooperate, but at the same time they have conflicting interests about the conditions of such cooperation (e.g., two or more agents have a common interest to trade, but conflicting interests on the price at which to trade). In other words, agents would like to reach an agreement rather than disagree; however, each agent would like to reach an agreement that is as advantageous as possible for itself [14]. Truthfulness of the participants is an essential element in bargaining. For this reason, a consistent branch of game theory is dedicated to the development of techniques, called *pricing mechanisms*, aimed at discouraging cheating among the players.

### 3.5.1 Nash bargaining

The Nash bargaining model applies to situations in which two individuals bargain over the partition of a fixed payoff: the set of possible agreement is the set of partitions whose sum is the total payoff. In case of disagreement, each player is penalized. The penalties are defined as the *disagreement point* of the game. The *useful payoff* for each player is thus defined as the difference between the payoff received in case of agreement and the disagreement point. The partition that maximizes the *product* of the agents' useful payoffs is referred to as the Nash bargaining solution or the Nash product [14].

### 3.5.2 Rubinstein bargaining

The Rubinstein bargaining game models a sequential bargaining, where offers and counteroffers are made by turns. In order to preclude an infinite negotiation, a cost is incurred in case the the agreement is delayed. Thus, at any given round of the bargaining, the power of a player is determined by the magnitude of this cost. In other words, at each additional round the size of the available payoff becomes smaller. The factor by which the payoff is decreased can be different for each player, and it is referred to as the player's discount factor. The Rubinstein bargaining shows a unique subgame perfect equilibrium, i.e., any offer made by a player should be equal to (or greater than) the discounted value that the opponent is able to get in the next round.

## 3.6 Bayesian games

The Bayesian games framework can be used to model scenarios in which the set of agents may be characterized by several behavior profiles (e.g., some agents can favor cooperation, while others may want to cheat). In particular, each agent does not know in advance the exact behavior that the other agents will apply to a certain situation, although it may know the whole set of possible types of behavior.

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The uncertainty about the characteristics of other players is modeled by introducing a set of possible states, called *player's types*, with their associated (guessed) probability of occurrence.

**Example 1** *In a bargaining situation for the allocation of a given payoff, the information disclosed by the agents (e.g., about the costs of operating their systems) may or may not be truthful, in order to take advantage and receive a greater share of the payoff; in such a case, Bayesian game model can be used to extract a profile of the other agents' actions.*

When choosing the strategy to be implemented next, a player should take into account the probability of occurrence of each of the other players' types, and thus their possible actions. Given that at least one player is not aware of the type —and thus of the payoff— of another player, a Bayesian game is a game with incomplete information [14].[2]

## 3.7 Multi-agent learning

An important line of research related with game theory is the development of learning algorithms in order to achieve a desired cooperative equilibrium. A learning algorithm is typically composed of three steps: *(i)* observation of the state of the environment; *(ii)* estimation of the *prospective utility*; *(iii)* update of the strategy [15].

The *best response* represents the simplest form of such algorithms: at each iteration, every agent applies the strategy which maximizes its utility. However, the outcome of such scheme is very sensitive to the initial condition. Convergence to an equilibrium is only guaranteed with particular types of utility functions, and the convergence to an efficient equilibrium cannot be ensured [15].

In [15] some relevant categories of advanced learning algorithms are introduced:

- *Fictitious play*: algorithms in this category are based on the estimation of the frequency with which other agents implement a given strategy. On the basis of its estimate, each agent can choose the best strategy. For some type of games (e.g., zero-sum games) this scheme converges to a Nash equilibrium.
- *Regret matching*: this class of algorithms is based on the minimization of the *regret from applying a certain strategy*, i.e., the difference between the utility of *always* applying that strategy and the utility achieved by implementing the current strategy.
- *Reinforcement learning*.
- *Stochastic learning*.

In order to overcome instability issues due to possible errors (i.e., nonrational strategies) during the learning procedure, these algorithms are commonly designed so that *the agents are allowed to choose*

[2]See §3.1 for the definition of games with incomplete information.

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*unintended strategies*, i.e., perturbing the reached equilibrium to study any opportunity of improvement. This kind of approach, called *learning by experimentation*, is receiving significant attention in game theory and multiagent learning, and showed good performances in communication applications [15].

### 3.8 Coalitional game theory

*Cooperative* game theory provides means of analyzing the behavior of self-organizing agents that can communicate and decide to *cooperate* in order to achieve some benefit. Also, cooperative game theory focuses on the design of mechanisms for the cooperation so to guarantee *fairness* and efficiency. However, since most research has been focused on noncooperative games so far, specialized tools are still needed for the design of such mechanisms, in particular those suited for the dynamical environments naturally encountered in engineering applications.

Under cooperative game theory, *coalitional games* and *Nash bargaining* provide tools for the analysis of situations in which the agents have to decide with whom to cooperate and under which conditions [15]. Nash bargaining focuses on the negotiation of the conditions for the cooperation within a given coalition (e.g., allocation of the payoffs among the members of a coalition). A coalitional game is uniquely defined by the pair $(\mathcal{N}, v)$, where $\mathcal{N}$ is the set of players,[3] and $v$ is the *value of a given coalition* in the game. The definition of this value determines the form and the type of the game [13].

The basic category of coalitional games is represented by games in *characteristic form*, where the value of a given coalition $\mathcal{C}$ depends only on the members that compose it, with no regard to how the rest of the agents are organized. In games with *transferable utility* (TU), a value is assigned to any possible coalition through a function $v : 2^{\mathcal{N}} \rightarrow \mathbb{R}$. The real value $v(\mathcal{C})$ associated with coalition $\mathcal{C}$ can be divided and transferred among its members (e.g., side-payments used to attract other players). The *payoff* $p_i$ is defined as the utility received by each agent $i \in \mathcal{C}$ after the division of $v(\mathcal{C})$. The vector of payoffs assigned to all the agents is referred to as the *allocation*.

For the members of any given coalition, an allocation is said to be *efficient* when it is obtained by splitting the entire value of the coalition, i.e., when the following condition holds:

**Definition 2 (Efficiency)**

$$\sum_{i \in \mathcal{C}} p_i = v(\mathcal{C}) \tag{2}$$

Furthermore, an allocation is said to be *individually rational* when it satisfies:

**Definition 3 (Individual rationality)**

$$p_i \geq v(\{i\}), \forall i \in \mathcal{C} \tag{3}$$

In other words, individual rationality implies that the payoff offered to any member of a coalition has to be at least equivalent to the payoff that can be achieved by playing independently. An allocation satisfying both (2) and (3) is designated as *imputation* [17].

[3] Referred to as the *agents* in the remainder.

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In different cases, the payoff is assigned *directly to the members* of a coalition, according to given rules (or possible constraints on the division of the utility). The individual payoffs cannot be redistributed (transferred) among the players. Such situations are modeled by coalitional games with *nontransferable utility* (NTU). Hence, the value of a coalition $\mathcal{C}$ is not expressed by a single real value; instead, $v(\mathcal{C}) \subseteq \mathbb{R}^{|\mathcal{C}|}$ is a *set* of allocations, each one relative to a given coalition's strategy.[4]

Games in characteristic form allow to model a wide spectrum of scenarios. In engineering applications, however, it is natural to encounter problems in which the value of a given coalition cannot be determined regardless of how the rest of the agents are organized. Games in *partition form* model such type of problems. Given a partition of the set of agents $\mathcal{N}$, i.e., a set of disjoint coalitions $\boldsymbol{\mathcal{C}} = \{\mathcal{C}_1, \dots, \mathcal{C}_l\}$, the value of a coalition $\mathcal{C}_i \in \boldsymbol{\mathcal{C}}$ is expressed as $v(\mathcal{C}_i, \boldsymbol{\mathcal{C}})$ (this value can be either TU or NTU).

The two game forms introduced so far do not consider any underlying communication infrastructure among the agents. Indeed, either in characteristic or in partition form, the value of a coalition does not depend on how the agents are connected. Coalitional games in *graph form* have been introduced to model situations in which the connections between the agents influence the utility that they can achieve. As the name suggests, these connections are modeled by means of a graph (either directed or undirected) whose nodes represent the agents. The value of a coalition $v(\mathcal{G}_\mathcal{C})$ is thus expressed on the basis of the topology of the edges connecting the agents of $\mathcal{C}$; depending on the scenario, the value $v(\mathcal{G}_\mathcal{C}, \mathcal{G}_{\mathcal{N}\setminus\mathcal{C}})$ can be a function of how the rest of the agents are connected [13].

## 3.9 Canonical coalitional games

Canonical coalitional games are in characteristic form (TU or NTU). The main feature of games belonging to this category is that *cooperation is always beneficial*. More specifically, the *superadditivity* property always holds:

$$v(\mathcal{C}_1 \cup \mathcal{C}_2) \geq v(\mathcal{C}_1) + v(\mathcal{C}_2) \tag{4}$$

which implies that the members of two disjoint coalitions $\mathcal{C}_1$ and $\mathcal{C}_2$ are able to achieve *at least* the same payoff allocation if participating in the coalition produced by the union $\mathcal{C}_1 \cup \mathcal{C}_2$. This, in turn, leads to the formation of the *grand coalition*, since the payoff allocation that can be obtained from $v(\mathcal{N})$ is at least as good as those obtained through the coalition of any possible subset of agents. Hence, the theory on canonical coalitional games focuses on two main problems:

- *stability*: how to find a payoff allocation guaranteeing that no agent would prefer to leave the grand coalition;
- *fairness*: it can be interpreted as a notion allowing to model the influence of "global goals" on the agents' behavior. The members of a coalition may care about receiving a payoff that is balanced with each one's contribution (thus, each agent care about both its own payoff and the others') [18].

[4]The notation $|\cdot|$ denotes the cardinality of a set.

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### 3.10 The core

The set of payoff allocations able to guarantee that no agent has an incentive to leave the grand coalition $\mathcal{N}$ to form a coalition $\mathcal{C} \subset \mathcal{N}$ is called the *core*. In particular, a payoff allocation $\mathbf{p} \equiv \{p_i\}_{i \in \mathcal{N}}$ belonging to the core satisfies the following two conditions[5]:

- *Efficiency* (budget rationality): $\sum_{i \in \mathcal{N}} p_i = v(\mathcal{N})$, i.e., the entire value of the grand coalition has to be shared among its members.
- *Group rationality*: $\sum_{i \in \mathcal{C}} p_i \geq v(\mathcal{C}), \ \forall \mathcal{C} \subseteq \mathcal{N}$, i.e., the payoff $p_i$ obtained by each member of the grand coalition has to be greater (or equal) to the one obtained by acting alone or within a smaller coalition.

A payoff allocation $\mathbf{p}$ belonging to the core is said to be *stabilizing* [17]. The core is not guaranteed to exist for all canonical games. When the core is empty, the grand coalition cannot be stabilized. The existence of the core in a TU game is related to the feasibility of the following LP problem:

$$\begin{aligned} \min_{\mathbf{p}} \ & \sum_{i \in \mathcal{N}} p_i \\ s.t. \ & \sum_{i \in \mathcal{C}} p_i \geq v(\mathcal{C}), \ \forall \mathcal{C} \subseteq \mathcal{N} \end{aligned} \tag{5}$$

Since the growth of the number of constraints in (5) is exponential in the number of agents, finding the existence of the core is an NP-complete problem. In practice, however, the search of imputations can be narrowed by considering those of most interest (possibly already known to be fair) in a given scenario.

Nonemptiness of the core is assured in *convex* games.

**Definition 4 (Convex game)** *A TU canonical game is convex if the following condition holds:*[6]

$$v(\mathcal{C}_1) + v(\mathcal{C}_2) \leq v(\mathcal{C}_1 \cup \mathcal{C}_2) + v(\mathcal{C}_1 \cap \mathcal{C}_2), \quad \forall \mathcal{C}_1, \mathcal{C}_2 \subseteq \mathcal{N} \tag{6}$$

or alternatively, for all $i \in \mathcal{N}$ and any pair of coalitions $\mathcal{C}_1 \subseteq \mathcal{C}_2 \subseteq \mathcal{N}$ such that $\mathcal{C}_1 \cap \{i\} = \mathcal{C}_2 \cap \{i\} = \varnothing$,

$$v(\mathcal{C}_1 \cup \{i\}) - v(\mathcal{C}_1) \leq v(\mathcal{C}_2 \cup \{i\}) - v(\mathcal{C}_2) \tag{7}$$

Definition (7) relates the convexity of the game to the marginal contribution of each agent $i$: in a convex game, the marginal contribution of any agent $i$ is *nondecreasing* w.r.t. set inclusion (i.e., the size of the coalition it joins). In other words, a game is convex if an agent's marginal contribution increases if it joins a larger coalition [19]. However, convexity is a strong condition, hard to find in real-world scenarios. *Balancedness* of a game is a weaker condition —which holds for a wide class of problems— that also guarantees a nonempty core, as stated by the Bondareva-Shapley theorem [17].

[5]The examples refer to a TU game. For NTU games the definition of the core is similar, although based on the individual payoff instead of their sum.

[6]The definition can be extended to NTU games as well.

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**Definition 5 (Balanced map)** *Let $\alpha : 2^{\mathcal{N}} \rightarrow [0,1]$ be a function that assigns a weight to each possible coalition in a set $\mathcal{N}$ of agents. The function $\alpha$ is said to be a* balanced map *if*

$$\sum_{\mathcal{C} \in 2^{\mathcal{N}}} \alpha(\mathcal{C}) \mathbf{1}\{i \in \mathcal{C}\} = 1, \quad \forall i \in \mathcal{N} \tag{8}$$

i.e., $\alpha$ is a balanced map if, for any agent $i$, the sum of the weights assigned by $\alpha$ to every coalition containing $i$ equals 1.[7]

**Definition 6 (Balanced game)** *A game $(\mathcal{N}, v)$ is said to be balanced if for any balanced map $\alpha$ it holds that*

$$\sum_{\mathcal{C} \in 2^{\mathcal{N}}} \alpha(\mathcal{C}) v(\mathcal{C}) \leq v(\mathcal{N})$$

**Theorem 7 (Bondareva-Shapley)** *A coalitional game has a nonempty core iff it is* balanced.

### 3.11 Shapley Value

As mentioned in §3.9, it is not uncommon to find the core to be an empty set or, vice versa, to be so large to make the choice of a suitable imputation a hard task. Furthermore, an allocation belonging to the core does not necessarily guarantee fairness to every agent (e.g., Bird's allocation rule).

Shapley tackled these issues by defining the conditions to obtain a unique payoff mapping, i.e., the *Shapley value* of the game $(\mathcal{N}, v)$. Any Shapley allocation $\phi$ must satisfy the following four axioms:[8]

1. *Efficiency*: $\sum_{i \in \mathcal{N}} \phi_i(v) = v(\mathcal{N})$.
2. *Symmetry*: given two players $i$ and $j$, if $v(\mathcal{C} \cup \{i\}) = v(\mathcal{C} \cup \{j\})$, $\forall \mathcal{C} | \mathcal{C} \cap \{i, j\} = \varnothing$, then $\phi_i(v) = \phi_j(v)$.
3. *Dummy*: if, for any player $i$, $v(\mathcal{C}) = v(\mathcal{C} \cup \{i\})$ holds $\forall \mathcal{C} | \mathcal{C} \cap \{i\} = \varnothing$, then $\phi_i(v) = 0$.
4. *Additivity*: let $u$ and $v$ be characteristic functions. Then $\phi(u + v) = \phi(u) + \phi(v)$.

The symmetry axiom assigns the same payoff to players that equally improve a given coalition, while the dummy axiom assigns a null payoff to a player which does not contribute when joining a coalition. The additivity axiom states the uniqueness of the mapping $\phi$ over the space of all coalitional games, relating the Shapley values of two different games characterized by $u$ and $v$ [13]. The formulation of the individual payoff assigned to player $i$ according to the Shapley value's mapping is:

$$\phi_i(v) = \sum_{\mathcal{C} \subseteq \mathcal{N} \setminus \{i\}} \frac{|\mathcal{C}|!(|\mathcal{N}| - |S| - 1)!}{|\mathcal{N}|!} [v(\mathcal{C} \cup \{i\}) - v(\mathcal{C})] \tag{9}$$

The weight factor of the marginal contribution $v(\mathcal{C} \cup \{i\}) - v(\mathcal{C})$ expresses the probability —for any agent $i$— of joining a given coalition $\mathcal{C} \subseteq \mathcal{N} \setminus \{i\}$, assuming that the agents form the grand coalition *in*

[7] The notation $\mathbf{1}\{\cdot\} : \mathcal{N} \rightarrow \{0,1\}$ designates the *indicator function*, which takes the value 1 if $i \in \mathcal{C}$, or 0 if $i \notin \mathcal{C}$ [20].

[8] The definition is given here for TU games. Definitions of the Shapley value for NTU games are also available.

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*a random order*. Thus, the value assigned by Shapley's criterion corresponds to the individual *expected marginal contribution*.

A Shapley allocation that also belongs to the core of a game combines the fairness properties of the Shapley value with the stability of the core. Note that for convex games, the Shapley value provides a closed-form expression of an imputation belonging to the core. However, in nonconvex games the Shapley value is generally not related to the core [17].

Despite the advantage of its closed form, the computational complexity of (9) increases significantly with the number of agents: alternative techniques for its computation can be found in the literature (the interested reader is referred to [13] and references therein).

## 3.12 The nucleolus

Another important allocation criterion for canonical games is the *nucleolus*. It consists in the allocation $\mathbf{p}$ (for the grand coalition) able to minimize the *dissatisfaction* of all the agents for all the possible coalitions in $\mathcal{N}$. The dissatisfaction is defined for each coalition $\mathcal{C} \subseteq \mathcal{N}$ as the excess

$$e(\mathbf{p}, \mathcal{C}) = v(\mathcal{C}) - \sum_{i \in \mathcal{C}} p_i \tag{10}$$

Consider the vector $\theta(\mathbf{p}) \in \mathbb{R}^{2^{\mathcal{N}}}$ containing the excess values for every coalition in the set of agents, arranged in nonincreasing order, relative to an imputation $\mathbf{p}$.[9] Then, the nucleolus is the imputation $\mathbf{p}^*$ providing the minimum dissatisfaction according to the lexicographic order, i.e.:

$$\theta(\mathbf{p}^*) <_{\text{lex}} \theta(\mathbf{p})$$

where the $<_{\text{lex}}$ operator designates the lexicographic order, defined as:

$$(e_1, \dots, e_{2^{|\mathcal{N}|}}) <_{\text{lex}} (e'_1, \dots, e'_{2^{|\mathcal{N}|}})$$

iff

$$\exists i \leq 2^{|\mathcal{N}|} \mid e_i < e'_i \wedge e_j = e'_j, \forall j < i$$

**Definition 8 (Nucleolus [17])** *The nucleolus of a game $(\mathcal{N}, v)$ is the lexicographically minimal imputation.*

As a consequence of the efficiency and group rationality conditions, only imputations with negative or null dissatisfaction belong to the core. If the core of a game is not empty then the nucleolus is in the core; following Theorem 7 this condition is surely verified in balanced games (which include convex games).

The nucleolus of a canonical coalitional game always exists and is unique. It is group and individually rational, and satisfies the symmetry and dummy Shapley's axioms. Moreover, it always belongs to the

[9] See §3.8 for a definition of imputation.

kernel of a game.[10] The nucleolus has so far been defined only for TU games, and its computation requires the solution of $\mathcal{O}(2^{\mathcal{N}})$ linear programs, each with $|\mathcal{N}|+1$ decision variables [17].[11]

### 3.12.1 An example

The authors of [17] analyze a coalitional setting of several wind power plants, in order to exploit the tendency of wind speed at different geographic locations to decorrelate with spatial separation. Since deviations from the contracts are penalized, the aim is to increase the benefit when placing offers on the day-ahead electricity market, thanks to the reduced variability of the aggregate production.

The value of a coalition is defined as the maximum expected profit deriving by the aggregation of the offer. The problem is relaxed by disregarding the structure and the dynamics of the power network, and considering all generators as connected to a common bus. It is shown that —due to the reduction in statistical dispersion— the grand coalition is always beneficial in such setting.

The resulting game is balanced, and so characterized by a nonempty core: the existence of a payoff allocation guaranteeing the stability of the grand coalition is thus ensured. Nevertheless, naïve mechanisms (e.g., equal distribution) cannot stabilize the grand coalition; since the game is not convex, the Shapley value does not provide guarantees on the stability either. In a balanced game, on the other hand, the solution provided by the nucleolus is surely in the core. However, its computational complexity hinders its use. On this basis, a method to fairly share the profit among the members of the coalition is proposed. It consists in an approximation of the nucleolus, minimizing the worst-case dissatisfaction for every coalition: for its computation, the solution of just a single LP is required. A brief synthesis of the results relative to one of the scenarios considered in [17] is provided in Fig. 2.

## 3.13 Power indices

Power indices express the influence of players on the formation of coalitions and on the outcome of the game [14]. A fair allocation rule can be based on (normalized) power indices. Widely used power indices are the one proposed by Shapley and Shubik (also known as Shapley value, see §3.11), the Banzhaf index, and the Holler-Packel index.

The Shapley value assumes an equal distribution of the probability of the order in which any agent joins all possible coalition. The Banzhaf power index assumes instead that all possible coalitions containing agent $i$ are equally probable. The measure of the power of agent $i$ according to the Banzhaf index is defined as:

$$\mathrm{BPI}_i = \frac{1}{2^{|\mathcal{N}|-1}} \sum_{\mathcal{S}\subseteq\mathcal{N}|i\in\mathcal{S}} (v(\mathcal{S}) - v(\mathcal{S}\setminus\{i\}))$$

[10] Given two players $i$ and $j$, the kernel of a game is the set of allocations such that the maximum dissatisfaction of a player $i$ belonging to any coalition not including player $j$ is equal to the maximum dissatisfaction of player $j$ belonging to any coalition without $i$.

[11] One decision variables corresponding to the payoff of each agent, plus an additional slack variable representing the excess for that coalition.

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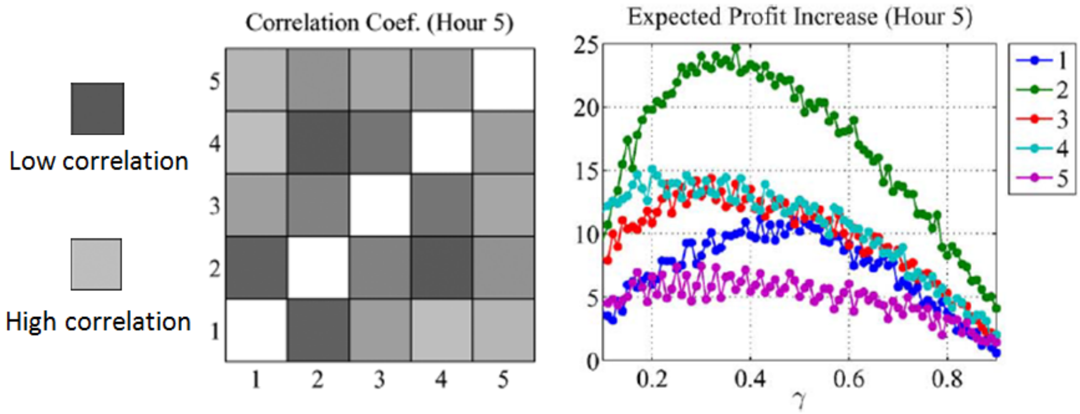


Figure 2: Results of the sharing method proposed in [17], obtained considering the aggregation of 5 wind farms. In the right plot, $\gamma = p/\mu_q$ is the ratio between the selling price and the penalty imposed on the deviations. The profit increases as the penalty price, and so the risk, increases. Notice how a higher share of the benefit is allocated to the wind plant showing the less statistically correlated production.

The Holler-Packel index is based as well on the marginal contribution of agent $i$, but it focuses on the coalitions whose allocations are in the core:[12]

$$\mathrm{HPI}_i = \sum_{\mathcal{S} \in \mathfrak{C}(\mathcal{N}, v)} \left( v(\mathcal{S}) - v(\mathcal{S} \setminus \{i\}) \right)$$

where $\mathfrak{C}(\mathcal{N}, v)$ denotes the set of coalitions whose payoff allocations constitute the core of the game (i.e., the best that can be achieved by the members of these coalitions, such that no one of them has an incentive to leave in search of a greater payoff).

**Example 9** *Suppose a game modeling a demand accommodation problem, where the demand cannot be satisfied by any participant supplier alone; a fixed, transferable reward is given to the coalition (of suppliers) capable of fulfilling the demand (*winning coalition*). It follows that to belong to the core of the game, a winning coalition will be eventually characterized by the minimal number of members necessary in order to fulfill the demand: in this way, the reward will be split among the least number of agents.*

Depending on the situation, some indices may be more suited than others. In particular, the Shapley value is based on the assumption that the agent commits to stay in the coalition it joins. On the other hand, the Banzhaf value is based on the idea that agents are free to join and leave any coalition.

The authors of [14] propose an index based on the "popularity" of the agent within the set of winning coalitions. In order to present it, a preliminary definition is given:

**Definition 10 (Minimal winning coalition)** *A* minimal winning *coalition is a minimal-sized coalition which, following the departure of any one of its members, is not able anymore to fulfill the requirements of the game (thus becoming a losing coalition)*

[12] See §3.10 for a definition of the core.

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Denote as $\mathcal{M}_i$ the set of coalitions through which a given agent $i$ achieves its maximum possible payoff.

**Example 11** *Consider a scenario in which a fixed prize is given to the winning coalition, and the individual payoffs are proportional to the contribution of each member to the aggregate yield of the winning coalition. Then any given agent will prefer to join a coalition providing the* minimal *yield necessary to win the game, such that its product would constitute a greater portion of the aggregate. In this way, given a fixed individual contribution, the payoff will be better.*

Then, for any minimal winning coalition $\mathcal{S}$ in the set $\mathfrak{C}(\mathcal{N}, v)$, a *preference index* $\omega(\mathcal{S})$ is defined as the total count of agents having coalition $\mathcal{S}$ in their set $\mathcal{M}_i$:

$$\omega(\mathcal{S}) = |\{i \in \mathcal{N} | \mathcal{S} \in \mathcal{M}_i\}|$$

Then the *Popularity power index* is defined as [14]:

$$\text{PPI}_i = \sum_{\mathcal{S} \in \mathfrak{C}(\mathcal{N},v)} \frac{\omega(\mathcal{S})}{\sum_{\mathcal{A} \in \mathfrak{C}(\mathcal{N},v)} \omega(\mathcal{A})} \mathbf{1}\{i \in \mathcal{S}\}$$

where $\mathbf{1}\{i \in \mathcal{S}\}$ is the indicator function, which equals 1 if $i \in \mathcal{S}$ and 0 otherwise. The Popularity power index relates the popularity of minimal winning coalitions an agent $i$ belongs to, to agent $i$'s influence on the outcome of the game. It can also be viewed as the probability that the selected winning coalition contains agent $i$.

## 3.14 Dynamic coalition formation

In this class of games the focus is directed on the evolution of the coalitional structure due to variations in the nature of the game (e.g., variations in the degree of coupling among different parts of the system, players that enter or leave the game in a plug & play setting). The purpose of the coalition formation process is to maximize the total utility (social welfare) in TU games, or to achieve a Pareto optimal payoff allocation in NTU games.

In general, the optimization of the composition of coalitions is an NP-complete problem, since it requires the evaluation of all the possible partitions of the set $\mathcal{N}$ of agents, whose number —known as the Bell number— grows exponentially with the number of agents.[13] Let $\mathcal{K}_s$ denote the set of possible coalitional structures (that is, partitions of $\mathcal{N}$) composed by $s$ coalitions. The cardinality of this set is expressed by the Stirling number of the second kind [21]:

$$|\mathcal{K}_s| = \frac{1}{s!} \sum_{j=0}^{s-1} (-1)^j \binom{s}{j} (s-j)^n \tag{11}$$

Then $|\mathcal{K}| = \sum_{s=1}^{|\mathcal{N}|} |\mathcal{K}_s|$ is the number of possible coalitional structures given a set $\mathcal{N}$ of agents [22].

As a consequence, a centralized approach for the optimization of the coalitional structure is impractical. In many cases the properties of the game (e.g., the way its value $v$ is defined) can be used as a guide

[13]For example, a set of 10 agents originates 115975 possible partitions.

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for reducing the computational complexity. Nevertheless, since coalition formation naturally involves several autonomous agents, a distributed approach is generally desired for the solution of such problem. Several techniques have been proposed so far, based on heuristic methods, Markov chains, set theory, bargaining and other negotiation algorithms from economics (the interested reader is referred to [13] and references therein).

The work of Apt and Witzel [23] focuses on the outcome of a coalition formation process resulting from the application of two rules, namely merge and split. The aim is to identify the conditions under which the outcome of the process is a unique coalitional structure, irrespective of the initial condition. The rules proposed in [23] are based on the comparison of different partitions of the set of players involved in a given merge or split operation, according to preference criteria such as Nash, utilitarian or leximin order (these ordering criteria provide advantageous properties, namely irreflexivity, transitivity, and monotonicity).

**Definition 12 (Merge [23])**

$$\{\mathcal{C}_1, \ldots, \mathcal{C}_k\} \cup \mathcal{P} \to \{\bigcup_{i=1}^{k} \mathcal{C}_i\} \cup \mathcal{P} \quad \text{iff} \quad \{\bigcup_{i=1}^{k} \mathcal{C}_i\} \triangleright \{\mathcal{C}_1, \ldots, \mathcal{C}_k\}$$

**Definition 13 (Split [23])**

$$\{\bigcup_{i=1}^{k} \mathcal{C}_i\} \cup \mathcal{P} \to \{\mathcal{C}_1, \ldots, \mathcal{C}_k\} \cup \mathcal{P} \quad \text{iff} \quad \{\mathcal{C}_1, \ldots, \mathcal{C}_k\} \triangleright \{\bigcup_{i=1}^{k} \mathcal{C}_i\}$$

In definitions 12 and 13, the symbol $\triangleright$ denotes the *local* preference operator. The rest of the agents not concerned with the transformation is denoted as $\mathcal{P} \equiv \mathcal{N} \setminus \bigcup_{i=1}^{k} \mathcal{C}_i$.

The application of supplementary basic rules can be studied to improve the convergence of the coalition formation process. For example, *transfers* (moving a subset of agents from one coalition to another) and *swaps* (exchanging subsets of agents between two coalitions) are considered in [24].

### 3.14.1 Preferences for TU games

Consider a coalitional TU game $(\mathcal{N}, v)$, where the value $v$ maps a given coalition to a nonnegative real value, such that $v(\varnothing) = 0$. Then consider two different partitions of a given set of agents, i.e., two *sets of coalitions* $\mathcal{A} \equiv \{\mathcal{C}_1, \ldots, \mathcal{C}_l\}$ and $\mathcal{B} \equiv \{\mathcal{C}'_1, \ldots, \mathcal{C}'_m\}$. For a TU game, the preference operator can be based on the value of the coalitions:

$$\mathcal{A} \triangleright \mathcal{B} \quad \text{iff} \quad v(\mathcal{A}) \triangleright v(\mathcal{B}) \tag{12}$$

where $v(\mathcal{A}) \equiv \{v(\mathcal{C}_1), \ldots, v(\mathcal{C}_l)\}$.

Several criteria can be used for the comparison of two given coalitional structures. Let $a = (a_1, \ldots, a_l)$ and $b = (b_1, \ldots, b_m)$. The following ordering criteria fulfill the properties desired for the preference operator, namely irreflexivity, transitivity, and monotonicity [23]:[14]

[14] Examples of different ordering criteria that does not satisfy such properties are also shown in [23], such as the elitist or the egalitarian order.

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- *Utilitarian order*:

$$a \succ_{\text{ut}} b \quad \text{iff} \quad \sum_{i=1}^{l} a_i > \sum_{i=1}^{m} b_i$$

- *Nash order*:

$$a \succ_{\text{Nash}} b \quad \text{iff} \quad \prod_{i=1}^{l} a_i > \prod_{i=1}^{m} b_i$$

- *Leximin order*:

$$a \succ_{\text{lex}} b \quad \text{iff} \quad \breve{a} >_{\text{lex}} \breve{b}$$

where $\breve{a}$ is the sequence of the components of $a$ arranged in nonincreasing order, and the $>_{\text{lex}}$ operator designates the lexicographic order, defined as:

$$(a_1, \dots, a_l) >_{\text{lex}} (b_1, \dots, b_m)$$

iff

$$\exists i \leq \min(l, m) \mid a_i > b_i \wedge a_j = b_j, \forall j < i$$

or

$$\forall i \leq \min(l, m), a_i = b_i \wedge l > m.$$

Notice that the Nash order implicitly promotes an equal distribution, since for a fixed $\sum_{i=1}^{l} a_i$, $\prod_{i=1}^{l} a_i$ is maximum when all $a_i$ are equal [23].

### 3.14.2 Individual payoffs

The preference criteria presented in §3.14.1 are based on the entire coalition's value. However, in practice each agent may base its preference on the *individual payoff* that it achieves by joining a certain coalition. In [23], the notion of *individual value function* is used in order to map the value $v(\mathcal{C})$ of a given coalition $\mathcal{C}$ to each agent's payoff. The individual value function $\phi_i(\mathcal{C})$ is assumed to be *efficient*, i.e.:

$$\sum_{i \in \mathcal{C}} \phi_i(\mathcal{C}) = v(\mathcal{C})$$

Let $\mathcal{A} \equiv \{\mathcal{C}_1, \dots, \mathcal{C}_l\}$ and $\mathcal{A}' \equiv \{\mathcal{C}'_1, \dots, \mathcal{C}'_m\}$ be two different partitions of the same (sub)set $\mathcal{S}$ of agents, i.e.,

$$\bigcup_{i \in \mathcal{A}} \mathcal{C}_i = \bigcup_{i \in \mathcal{A}'} \mathcal{C}'_i = \mathcal{S} \subseteq \mathcal{N} \tag{13}$$

Furthermore, let $\Phi(\mathcal{A})$ be the vector of the allocations of the coalitions in the partition $\mathcal{A}$. According to [23], no general relation can be found between orderings based on $v(\mathcal{C})$ and orderings based on $\phi(\mathcal{C})$. An exception to this is represented by the utilitarian order, since by definition (see §3.14.1) it compares the sum of the payoffs, that in turn, following the efficiency assumption (13) on $\phi(\mathcal{C})$, coincides with the coalitional value $v(\mathcal{C})$.

The *Pareto order* can be used as a preference criterion for two different set of allocations $\Phi(\mathcal{A})$ and $\Phi(\mathcal{A}')$. Notice that in this case the comparison is on the payoff achieved by each agent in the set $\mathcal{S}$:

$$(\phi_1, \ldots, \phi_n) \succ_P (\phi'_1, \ldots, \phi'_n) \quad \text{iff} \quad \forall i \in \{1, \ldots, n\},\, \phi_i \geq \phi'_i \wedge \exists j \,|\, \phi_j > \phi'_j \tag{14}$$

where $\succ_P$ designates the Pareto order, and $n$ is the number of agents in $\mathcal{S}$. The Pareto order satisfies the transitivity, irreflexivity and monotonicity properties desired for the preference operator [23].

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# 4 Literature survey

*The literature survey available in this section is provided in order to trace an ideal path from the optimal model partitioning to the distributed control schemes whose principles happen to be close to the idea of coalitional control. Following the same path, the survey concludes with a presentation of several works relating the classic notions of game theory to the control engineering field.*

## 4.1 Model partitioning

Model partitioning consists in the choice of the subsets of the global state and input variables to be assigned to each agent involved in the control of a system. It is regarded as an unsolved key problem for the application of distributed control strategies to large-scale systems. Diverse approaches have been published so far (see, e.g., [25, 2, 10, 26, 27]). A common assumption is that the overall model of the system is available [28]. It is clear that a general systematic methodology is hard to define, due to the manifold nature (subsystems' interactions, time-scale, communication constraints, privacy concerns) of controlled systems, so that ad hoc approaches are frequently proposed.

According to [29], system decomposition can be either *horizontal* or *hierarchical*. The first type relates to the physical structure of the system, while the second type is based on the nature of the process, on its characteristic time scales, and on the control objectives. In general, hierarchical decompositions provide the designer with superior flexibility [28]. Furthermore, a hierarchical structure offers the advantage of a greater independence between agents and between neighborhoods [9].

An algorithm for the optimal partitioning of large-scale systems aimed at the implementation of MPC control on partitions (subsystems) of manageable size is presented in [25]. The approach is based on the representation of the whole system as a directed weighted graph, whose connectivity is measured on the basis of the Hankel norm, resulting from the controllability and observability Grammians of the system. The algorithm is composed of two steps: *(i)* grouping of actuators to be controlled by each agent, and *(ii)* choice of the output measures to be communicated to the agents, as a result of the previous grouping. A compromise between two criteria is used as a guide for the grouping of the actuators: *(i)* an open loop criterion, maximizing the controllability of the system (after the grouping) measured through its Hankel norm, and *(ii)* a closed loop criterion, minimizing the performance degradation due to partitioning, measured by the MPC's cost function.

Finally, the answer to the question "*which output measures must be known by each agent?*" is obtained through the identification of the matrix representing the sensitivity of the closed-loop control action to each output measurement.

More often than not, large-scale systems are characterized by multi-scale dynamics, e.g., slower overall dynamics arising out of a group of subsystems with fast dynamics. In [2], a structural analysis for the partitioning of the system model is presented, oriented to the implementation of multi-scale distributed estimation and control. The challenge is on the management of a distributed database (a concept that

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can be related to *cloud computing*) of systemwide measures to achieve a globally optimal—yet scalable—control. According to the authors of [2], several decentralized schemes proposed in the past lack in efficiency, due to either assuming a priori bounds on the coupling between subsystems, or considering the worst-case interactions. A definition for *distributed system* under a data fusion perspective is given (based on a reference therein): "a data processing system in which all the information is treated locally without the presence of a central processing site". Within such system, data fusion nodes obtain local observations, share relevant information with other nodes, and then compute a *globally optimal* state estimate and input sequence.

The attention is drawn towards the relationship of the model decomposition with the sampling time: as a general rule, the sampling time needs to be sufficiently small in order to preserve the (possibly sparse) structure of the system under analysis [2]. Whenever this would require an unachievable sampling time, a hierarchical multi-rate control structure can alternatively be implemented, so that the measures are obtained on a interval that is a multiple of the sampling time.

The authors of [2] list as key points in the partitioning of large-scale systems the similarity of the resulting decomposition to the actual plant topology, the availability of computational resources, the computational load at each node, and the overhead due to communication. These may constitute conflicting goals at the moment of searching for the optimal partition. Indeed, excessive partitioning of the system may reduce the computational requirements at each node, yet at the expense of an increased communication load. This in turn limits the possibilities of a parallel implementation of the control algorithm. Similarly, while a small sampling time allows to obtain a model that reflects the actual structure of the system, it will result in high communication rates between nodes—as well as in shorter time available for computation. Whenever possible, the decoupling of different parts of the system can be achieved by using buffers (e.g., storage devices).

In [30], the interaction among subsystems resulting from the partition of a global linear state-space model is viewed as a multiplicative output uncertainty. The authors propose a coupling metric, based on the open-loop evaluation of the uncertainty bound.

## 4.2 Cooperative distributed control

In the distributed control literature, two categories are delineated according to the degree of "selfishness" of the objective function used: *noncooperative* control, where only the local objective is pursued by each agent, and *cooperative* control. The importance of *cooperation* among MPC control agents is underlined in [8]. The authors of this work show that the exchange of information among subsystems provided with interaction models does not constitute a sufficient guarantee for closed-loop stability. The reason for this is the *competition* among agents, arising from the pursuit of conflicting objectives. In the framework proposed by [8, 6, 31], the objective function of each agent is modified in order to favor systemwide objectives,[15] thus guaranteeing (under given assumptions) nominal stability. The downside of this technique is the requirement of a high communication rate.

Some open questions regarding distributed MPC control are stressed in [8]: *(i)* how to exploit the system's structure,[16] *(ii)* how to deal with coupled constraints, *(iii)* how to make the control scheme robust against disruptions in the communication, *(iv)* what is the importance of providing subsystems with plug-and-play capabilities? Also, an important issue to take into account when designing control schemes for large-scale systems is to avoid the need of a complete redesign and reimplementation of the already present control infrastructure (i.e., trying to keep the transition as smooth as possible).

An iterative algorithm for a cooperative distributed MPC controller is presented in [8]: besides the standard definition of a quadratic objective for each subsystem, it is assumed that subsystems are only subject to local constraints. An input trajectory is communicated at each iteration, and then combined with the previous optimal solution. The proposed scheme is compared with centralized, decentralized, and *communication-based* control. According to this strategy, each agent is provided with a model of its subsystem and of the effect that other agents' actions produce on their dynamics and objective. The difference with the proposed cooperative strategy resides in the objective function: only the local objective is expressed in the communication-based MPC's cost function, while a copy of the global one is available to each agent in the cooperative MPC. Two possible behaviors have been shown in simulation: either move towards a *Nash equilibrium* or towards a *Pareto optimal* solution. The first is defined as a point satisfying the optimality condition for each local agent acting as an independent entity (for a more detailed definition, see §3.4). The second case is obtained through the equal combination of all the agents' objectives, and coincides with the solution of an equivalent centralized MPC.[17] The authors of [8] show that, regardless of the knowledge about the global dynamics of the system that may be available to each agent, the closed-loop behavior can range from stable and almost optimal solutions by both communication-based and cooperative schemes, to far-from-optimal or even unstable Nash equilibria. This sensible difference in performance is likely to be detected when dealing with strongly coupled subsystems.

In order to relax the heavy communicational requirement typical of iterative distributed schemes, the strategy presented in [31] admits the injection of suboptimal control actions to input-coupled subsystems.[18] *Sparsely coupled* input constraints are handled through the reassignment of the corresponding decision variables among the agents. A method for the derivation of a distributed model is suggested, which is based on the Kalman canonical form representation of the linear state-space model for each input-output pair [34].

The work of [31] is extended in [9] to a hierarchical framework, allowing the asynchronous update among

[15] Another way to view it is that each agent—by using a global cost function—weighs the effect of its actions upon all subsystems.
[16] Notice that some topologies are more problematic than others (e.g., cycles).
[17] Provided the absence of coupling constraints [32].
[18] The algorithm is based on suboptimal control theory, see [33, Chap. 6].

different neighborhoods. The neighborhood is defined in [9] as the set of (input-coupled) subsystems for which the interaction has no time delay. The *upstream neighborhood* is defined as well, as the set of subsystems whose input has an effect on the state of a given subsystem after a one-step delay. According to these definitions—and through a change of variables that assigns the dynamics of the inter-neighborhood coupling to the *neighborhood leader*[19]—the communication requirement of the algorithm is reduced to a synchronous exchange with neighbors, and to an asynchronous exchange with upstream neighborhoods' leaders.

The authors of [8] point out the importance of the system's topology as a guideline for the reduction of the communication overhead characterizing cooperative MPC schemes. Typically only adjacent subsystems are directly coupled: most interactions beyond neighboring subsystems are seen through intermediate subsystems. Furthermore, in system characterized by a single "product" flow (e.g., chemical plants, roads, water distribution networks) a fundamental distinction can be made in modeling the interactions involving each subsystem: influence of *upstream* subsystems—included in the distributed dynamic model—and effect of each agent's actions on *downstream* subsystems. Indeed, undesired downstream effects may be reduced by expanding each agent's cost function with a term modeling downstream subsystems' dynamics.

Trodden and Richards have extended the distributed cooperative formulation to *dynamically uncoupled* subsystems affected by bounded disturbances and *coupled constraints*. In [35] they present a non-negotiating cooperative algorithm. Based on robust-tube local feedback MPC [36, 37, 38, 39], the algorithm provides robust feasibility and stability guarantees. In particular, any agent's objective function can consider terms (partially) covering system-wide goals.

To cope with coupled constraints, only one agent is allowed to optimize its tube feedback law at each step, while the others implement their shifted (previously optimized) sequence. A much slower communication rate is achieved w.r.t. iterative/bargaining algorithms (such as [31, 40]). For robust constraint satisfaction, the shifted inputs implemented in the rest of the system are considered in the optimization problem solved by each agent; furthermore, the algorithm involves the computation of optimal actions for the subsystems belonging to the so-called *cooperating set*, although these sequences are not communicated. The rationale behind such hypothetical control inputs is to optimize the individual strategy considering what others may be able to achieve. Indeed, this approach is intended to promote cooperation among the agents, by *indirectly favoring the best plans for everyone within the cooperating set*. Notice that, in such scenario, the members of a cooperating set have to share their dynamic models. After the optimization, the agent's new sequence is broadcast[20] along with the terminal robust invariant set, necessary for the satisfaction of coupled constraints. The authors of [35] point out that the

[19]The change of variable provides the advantage of not having to share input trajectories between different neighborhoods [9].

[20]Actually, this information is needed by the members of its cooperating set: the rest of the agents only need the planned outputs involved in the coupled constraints.

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cooperating set is not necessarily restricted to the subsystems involved in the coupled constraints. The order in which the agents optimize can be either fixed or updated on-line; also, thanks to the robust tube feedback formulation, there can be steps with no update (by shifting all input sequences). This makes such algorithm interesting where agent failure or communication disruption are of primary concern.

The strength of the cooperation can be tuned acting on *(i)* the composition of the cooperating sets and *(ii)* the weights of the objectives relative to the other members of the cooperating set in each agent's cost function. The choice of these parameters is beyond the scope of [35]. Notice that due to the fact that only one agent is allowed to optimize at each time step, the performance would not be equivalent to that of centralized control even if the cooperating set coincides with the whole set of agents.

Focusing their study on the robustness properties of distributed control schemes, the authors of [28] review the fundamental strategies available in the literature. On the same line of [8], the attention of the reader is brought upon the following key issues: *(i)* the selection of optimal control structures, *(ii)* the coordination strategies among the controllers and *(iii)* the robustness of distributed schemes to model errors. This last point is of great importance when dealing with predictive control strategies. Indeed, linear models—whose validity is limited around the identified range of plant operation—are the most used in practice. Furthermore, probability of data loss (due to, e.g., network or sensor failure) increases the risk of inaccuracies.

Along with "horizontal" schemes, hierarchical structures are proposed for the distributed control of large-scale systems. Indeed, hierarchy provides larger flexibility in shaping the controller to the heterogeneous composition of many large-scale system, e.g., different sampling times, asynchronous operation of the parts. Aske et al. propose in [41] the *coordinator MPC* strategy, where the most relevant variables of the system are controlled by a central coordinator, and a set of decentralized controllers complete the control action by responding to the actions of the coordinator.

## 4.3 Towards coalitional control

In §4.2, *cooperating sets* have been introduced as defined in [35]. A possible method for the on-line update of these cooperating sets is presented in a previous work by the same authors [32], based on the identification of the active coupling constraints. As in [35], the proposed strategy addresses the robust distributed MPC control of dynamically-decoupled subsystems, that are nonetheless coupled through constraints and subject to bounded disturbance. At each time step, a graph representing the coupling of the subsystems is updated by identifying the constraints that were active at the previous time step. The cooperating sets are formed by those subsystems that are connected through paths in the graph. The study in [32] shows that the optimal cooperating set is not necessarily the set of *directly* coupled subsystems. Moreover, numerical examples show that increased cooperation (w.r.t. a greedy strategy) not always translates to a gain in performance; indeed, in some cases it may even lead to a performance loss. This emphasizes the importance of the criteria upon which the cooperation is based.

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A different approach is presented by Valencia Arroyave in [40], where the distributed MPC problem for dynamically-coupled[21] subsystems is analyzed under a *dynamic bargaining game* perspective. The classic axiomatic bargaining game framework considers a *static* decision environment. Few recent proposals extend the theory to cover dynamic decision environments.[22] These works focus on the coalition as the objective of the bargaining, in order to implement a coalition-wide solution (the same for all members). However, the application of the same control input by all the agents in a coalition is likely to provide poor performances—if not infeasible—in a controlled system. Furthermore, in distributed MPC problems the formation of coalitions is intended as a mean to improve the performance: as such, it should be the consequence—and not the objective—of the bargaining procedure [40]. Thus, the original axiomatic bargaining game theory is extended in [40] in order to consider features such as dynamic decision environment and infinite plays (reflecting system's state dynamics and time horizon, respectively) that characterize control problems. In particular, the discrete-time dynamic bargaining game is viewed as a sequence of static bargaining games.

The proposed algorithm minimizes a cost function expressing the common goal of all subsystems, obtained through a combination of their individual goals. Notice that any individual objective function depends on the actions taken at global level. Therefore, the success of each agent in reaching its goal will depend on the choices made by the others. Any agent accepts to be part of a coalition only if the associated benefit is greater than the performance expected by acting independently. In particular, the satisfaction of a minimum individual performance index is guaranteed by the definition of a *disagreement point* as the threshold of maximum allowed loss of performance in case of cooperation. This disagreement point is not fixed: rather, it is expected to vary according to the state of the system. One agent's disagreement threshold is decreased whenever it decides to cooperate, and increased if it does not (fostering a later participation); in this way, the disagreement point tends to the optimal expected value of the objective function. The algorithm proposed in [40] is a non-iterative, non-cooperative process based on the Nash's two-person game negotiation model. The basic steps are:

1. All the agents exchange their state and input vectors and their expected utility.
2. With the received data, each agent is able to solve an optimization problem to determine a feasible control sequence minimizing a convex combination of all individual objective functions.
3. If a feasible sequence is found—such that the expected utility can be achieved—the agent accepts cooperation. Else, the shifted (previously-computed) control sequence is applied.
4. The control actions to be applied are communicated together with the updated disagreement thresholds.

[21] In the remainder, with *dynamically-coupled* we designate subsystems whose state depends on both state and input of some other subsystems.
[22] See references in [40].

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Notice that this algorithm requires the agents to share their objective function as well as the value of the state and input vectors needed to compute the individual components in the global utility. Since the decisions of the agents are based on such system-wide knowledge, the algorithm is able to achieve Pareto-optimal solutions.

According to the author of [40], the problem of determining the disagreement threshold is fundamental, and still open. Also, the hierarchical partition of a system is cited as another important open line of research.

One of the fundamental features of coalitional control is the effort towards a more efficient use of the system's infrastructures. The most critical one in the execution of distributed control algorithms is the data network. Some solutions were explored so far on this line. In [42], the exchange of information among agents is reduced through the use of dynamic models of coupled subsystems—along with the local one—in order to provide each agent with an estimate of the evolution of its neighbors.[23]

The coupling between subsystems is assumed to act only through the state (inputs are decoupled), and the control is performed by means of a distributed linear feedback law. An analogous technique is used in [12]: considering the effect of input coupling as constant disturbances—on the basis of the slow dynamics of the plant under study—the interactions between subsystems are estimated with a Kalman filter.

An $H_\infty$ robustness index is proposed in [43] to evaluate the performance of distributed MPC schemes—characterized by different degrees of coordination—w.r.t. model errors. The formulation of a multi-objective MINLP based on this index allows to seek a tradeoff between global robust performance and the degree of connectivity among agents.[24] For $N$ subsystems, the MINLP considers $N(N-1)$ binary variables to modify the structure of the controller, corresponding to all the possible connections among the subsystems. Results of simulations in [43] show how controllers characterized by dense connectivity are more sensitive to errors in the interaction models. When the uncertainty in the model is significant, a fully decentralized structure can provide more robustness than a centralized controller.

## 4.4 Analysis of coalitions

Saad et al. study the employment of coalitional game theory in engineering applications, with particular attention to wireless networking. Three main branches of coalitional games are distinguished: *(i)* canonical, *(ii)* coalition formation and *(iii)* coalitional graph games [13].

Coalition formation games model scenarios in which the *network structure* and the *cost for cooperation* play a major role: hence (as opposed to canonical games) the formation of a coalition is not always

[23] The concept of *neighborhood* in distributed control is generally associated with dynamic coupling, and not necessarily imply physical adjacency.

[24] Connectivity is considered in [43] in terms of knowledge of (state and input) interaction model. Different criteria may be used for the evaluation of the degree of connectivity, involving, e.g., conformity to the system's topology, cost of communication.

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beneficial. The theory about coalition formation games focuses on issues like: *which coalitions will form?*, *what is the optimal coalition size?*, *which methodologies can be used to study the properties of the resulting structures?*

The main features of a coalition formation game are [13]:

- The game can be in either characteristic or partition form (TU or NTU).
- In general, the *superadditivity* property does not hold.
- The formation of a coalition brings benefit to its members; however, gains are limited by the costs of forming the coalition. As a results, seldom the *grand coalition* is the optimal structure.
- Environmental changes can affect the network's topology (e.g., variation of the number or the strength of the agents).
- A given structure may be imposed by some external factor (e.g., physical constraints).

Coalition formation games can be also classified in *static* and *dynamic*. While in the first case a structure is imposed on the coalitions by some external factor (and the objective is to study this structure), the second type is concerned with the analysis of the formation of coalitions through *interaction* between the agents. Moreover, the structure may evolve according to some externalities. Properties of this dynamical structure and its adaptability to the environment is the object of the research about dynamic coalition formation games. Unfortunately, the availability of formal rules and analytical concepts is limited to canonical games (i.e., those where the grand coalition yields most benefit). Problems of dynamic coalition formation are generally application-specific, which contributes to the difficulty in the development of tools for their analysis [13].

The presence of a coalitional structure different than the grand coalition hinders the use of concepts such as the core, the Shapley value, and the nucleolus, typical of canonical games. This fact was first pointed out by Aumann and Drèze [44] for a static coalition formation game. Through the redefinition of the *group rationality* concept with that analogous but restricted to a given coalition of *relative efficiency*,[25] the definitions of the core, the Shapley value and the nucleolus were extended to a static coalition formation game in characteristic form with TU. However, the results in [44] clearly showed that the complexity of the problem increases sensibly with NTU, games in partition form, and dynamic coalition formation, especially when the solution has to be computed in a distributed manner [13]. This motivates the application-oriented solutions proposed in recent literature, such as [23, 45].

One of the fundamental problems studied in the coalitional game theory is the fair allocation of benefit among the members of a coalition. As discussed in §3, this is in general a complex problem to solve, which hinders its real-time application in dynamic environments. Nevertheless, a new branch in the

[25]According to this concept, the value of a given coalition (within a coalitional structure) must be divided among its members

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literature —where control engineering and game theory meet— is rapidly growing, addressing so far the management of wireless networks [13] and of the smart grid [17, 15].

The aggregation of renewable-energy plants in the electricity market for their participation in the electricity market is analyzed as a canonical coalitional game in [17] (see also §3.12.1). The main objective of the aggregation is the reduction of the variability in the production due to the unpredictability of renewable energy sources, exploiting the tendency of wind speed at different geographic locations to decorrelate with spatial separation. A penalty mechanism acting on deviations from the contracted energy provision characterizes the two-settlement market (day-ahead and real-time markets) where the aggregated production is offered. In [17], the selling price is assumed fixed and known, whereas the penalty prices (for both excess and lack of supply) are modeled as random variables. Restricting the analysis to energy sellers connected to a unique bus in the electrical network[26] the authors show that sharing the risk through the formation of the grand coalition is always beneficial.[27] The contract of energy supply to be offered in the market is based on the expected profit, which in turn depends on the expected unbalance computed by modeling the wind power yield as a random process.[28]

The focus of [17] is on finding a fair mechanism for the allocation of the benefit among the producers. Intuitively, those individuals who contribute to a larger reduction in CVaR should receive a greater share of the benefit obtained by the cooperation. It is shown that such scenario is modeled by a *balanced* game, characterized by a nonempty core. However, since balanced games are a superset of convex games, the allocation corresponding to the Shapley value is not guaranteed to be stabilizing. A fair and stabilizing allocation is provided by the nucleolus (see §3.12). Unfortunately, its computation requires the solution of $\mathcal{O}(2^N)$ LP optimizations. Nevertheless, the concept of nucleolus is used in [17] as a base for the formulation of a single linear programming optimization that minimizes the *worst-case* dissatisfaction for all possible coalitions. In the case of balanced games, the solution to this approximated problem is still guaranteed to lie in the core.

Naturally the actual realization of the profit is variable and different from the expected one, on which the contract and the allocation mechanism are based. Indeed, the allocation of the actual daily profit —obtained through the minimization of the worst-case dissatisfaction based on the expected values— may cause some producers to defeat the grand coalition in the short run (some producers may even have to pay, instead of receiving a payment). However, in the long run, the allocation is shown to converge to the expected one.

The authors of [17] remark that an alternative formulation, based on noncooperative games, can be considered to tackle the same problem. In this case, the problem consists in how to allocate the

[26]The producers are assumed to be located close enough to allow their participation in a common market, and transmission line dynamics are neglected.

[27]In such scenario, the agents would be indifferent to acting independently or within a coalition only in case the random wind processes are perfectly positively correlated.

[28]In particular, the benefit is attributed to the reduction of statistical dispersion as measured by the conditional value-at-risk (CVaR) — a function of the gap between the unconditional mean and the mean in the $\gamma$-probability tail.

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imbalance cost to the responsible producer, as well as asserting the existence and the properties of the Nash equilibrium. Finally, the efficiency loss w.r.t. to the cooperative solution could be measured through, e.g., the *price of anarchy* or the *price of stability* [17].

A monograph on coalition formation games can be found in [45]. In [46], the endogeneous formation of coalitions is studied. In particular, it is assumed that players are divided into several disjoint sets, or *coalition structures*. A value for each possible coalition structure is calculated and the stability of the players within the coalition is studied. Likewise, it is considered that agents may want to cooperate in some situations and to act separately in others, depending on how the rest of agents are organized. Hence, both coalition value and stability of a given configuration depend on the particular coalition structure considered.

In [47], the definition of stability in a coalition formation game is generalized through the Graph Model for Conflict Resolution and an algebraic formulation is provided in order to allow computer implementation. As a case study, a conflict over the proposed exportation of bulk water from Lake Gisborne (Canada) is analyzed. The same theoretical graph model framework is used in [48] for coalition stability analysis. Several definitions of stability (Nash, general metarational, symmetric metarational, sequential) are extended into a coalitional setting. A dispute due to the pollution of an aquifer by a chemical plant in Canada is used as a case study.

In [49], a two-stage coalition formation process is analyzed. In the first stage, links between pairs of agents are formed whenever they both agree and in the second agents negotiate about the payoff they should receive. This allows to model the process of cooperation structure formation as a game in strategic form and to obtain predictions of the expected cooperation structure.

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# 5 Game theory and Dymasos case studies

*The content of this section is a first attempt to explore the application of game theory to the smart grid. In order to reveal possible lines of research for the development of solutions for the Dymasos case studies (in particular the electrical grid case study by HEP and the EV charging case study by AYESA), the open issues evidenced by studies concerning the management of microgrids and storage devices connected to the grid are briefly presented.*[29]

## 5.1 Introduction

Smart grids constitute an exhaustive example of heterogeneous large-scale system. They encompass advanced power, communication, control and computing technologies. The relevant problems associated with the control of smart grids are: *(i)* microgrid systems, *(ii)* demand-side management, and *(iii)* exchange of information among the "smart" components of the grid.

The potential of the use of game theoretical tools in the control of smart grids is discussed by Saad et al. in [15]. Different applications in the smart grid's domain can be efficiently analyzed and managed with the aid of specific concepts of game theory, in particular those related to economical factors such as energy markets and dynamic pricing. The noncooperative games framework can be useful, e.g., to devise pricing schemes for demand-side management. Cooperative game theory can be applied to study a method for relaying the information in order to improve the efficiency of the communicational infrastructure. An important challenge faced in the design of such control systems for smart grids is the possibility of cheating (e.g., in energy market auctions). Methods to deal with this, e.g., strategy-proof auctions, have been recently proposed in the literature (see references in [15]).

## 5.2 Microgrids

A microgrid is defined as a heterogeneous network of distributed energy sources located at the *distribution side* of the electricity network, that can provide energy to a restricted geographical area. A microgrid may operate either together with the main grid, or in an autonomous fashion.

Power generation within microgrids is mainly based on small production from renewable sources or combined heat and electricity production (e.g., photovoltaics, wind turbines, $\mu$CHP generators). Furthermore, the availability of storage facilities (e.g., electric vehicles' batteries) and flexible demand is foreseen. Being characterized by such intermittent and unpredictable generation pattern, microgrids may need to compensate for the missing energy by buying it either from other microgrids with exceeding power production, or from the main grid. In [15], a game-theoretic formulation of the energy exchange problem among microgrids and the main grid is proposed, with the aim of reducing the energy losses in the lines. Such cooperative energy exchange mechanism has the advantage of improving the autonomy

[29] This section is mainly based on the survey [15].

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of microgrids w.r.t. the main grid, as well as reducing the losses over the distribution lines by promoting local energy trade among neighboring microgrids. As an introductive example of the application of game theoretical concepts in the management of microgrids, a description of the method proposed in [15] is provided in §5.2.1.

### 5.2.1 Cooperative energy exchange among microgrids

Consider a distribution network composed of a substation connected at one side to the main grid, and at the other side to a set $\mathcal{N} \equiv \{1, \dots, N\}$ of microgrids. Each microgrid $i \in \mathcal{N}$ is composed by a group of small generation devices as well as some consumers, distributed over a small area.

A variable $Q_i$ represents the difference between generation and demand of microgrid $i \in \mathcal{N}$. Since it is assumed that the generation is based on renewable sources and the demand is unpredictable, the variable $Q_i$ is considered to be random. At any given time, each microgrid is in either of the following states: *(i)* $Q_i > 0$, i.e., it has an energy surplus, or *(ii)* $Q_i < 0$, i.e., it needs to acquire power from an external seller.[30]

In the absence of any kind of storage, and without cooperation, each microgrid compensates for its energy need $Q_i < 0$ by acquiring it from the main grid. The consequent transfer is accompanied by an energy loss over the distribution lines. This energy loss represents the *noncooperative utility* of the microgrid:

$$u(\{i\}) = -w_i P_{i0}^{\text{loss}} \tag{15}$$

where $w_i$ is the price paid by $i \in \mathcal{N}$ per unit of power loss, and $P_{i0}^{\text{loss}}$ is the power lost in the transfer between microgrid $i \in \mathcal{N}$ and the substation.[31] The value of $P_{i0}^{\text{loss}}$ depends fundamentally on the distance from the substation, on the amount of power $Q_i$ which is transferred.

Now suppose that, for reasons like, e.g., constrained exchanges at the distribution layer (so to minimize losses at the transformer), convenient geographical location of the microgrids, the microgrids in a set $\mathcal{C} \subseteq \mathcal{N}$ are willing to cooperate by instituting a local energy trade market. Collaterally this will increase the autonomy of this coalition of microgrids. The low-level coordination protocol is not discussed in [15]; in general, the operations can be managed either by a multi-agent platform, or by a single entity (e.g., the owner of the microgrids).

In the following, a coalitional game is formulated among the microgrids in $\mathcal{C} \subseteq \mathcal{N}$. This group can in turn be divided into two categories: *sellers*, $\mathcal{C}^S \subset \mathcal{C}$, and *buyers*, $\mathcal{C}^B \subset \mathcal{C}$, such that $\mathcal{C}^S \cup \mathcal{C}^B = \mathcal{C}$. Microgrids belonging to either group can exchange energy with microgrids belonging to the complementary group, as well as with the substation.

The utility of a coalition $\mathcal{C} \subseteq \mathcal{N}$ can be a function of the members of $\mathcal{C}$ and of the way sellers and

[30] Microgrids whose generation can match exactly their demand are not involved in the problem, since they do not participate in the energy exchange.

[31] The substation is denoted as 0. The minus sign is inserted to get a maximization problem.

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buyers are matched (denoted as Π):

$$u(\mathcal{C},\Pi) = -\left(\sum_{i\in\mathcal{C}^S, j\in\mathcal{C}^B} w_i P_{ij}^{\text{loss}} + \sum_{i\in\mathcal{C}^S} w_i P_{i0}^{\text{loss}} + \sum_{j\in\mathcal{C}^B} w_i P_{j0}^{\text{loss}}\right) \tag{16}$$

where the first term expresses the losses on the local energy transfer between sellers and buyers, and the remaining two terms express the losses on the transfers with the substation. The utility function (16) can be used as a criterion for coalition formation, and as an objective for the optimization of the matching between buyers and sellers (such matching problems may be addressed with the aid of *auction theory*).

According to Saad et al., a suitable model for the scenario described in this section is the *double auction*. The strategy chosen by each member of $\mathcal{C}$ will depend upon the price at which it is willing to sell (buy) a given quantity of energy. Quantity and price thus constitute the manipulated variables for the optimization of the objective function, determining the equilibrium of the market.[32] The payoff of each microgrid $i \in \mathcal{C}$ can be computed according to (16) on the basis of the final values of these variables. The allocation will in turn determine the stability of a given coalition.

The complete solution of the cooperative energy exchange problem involves the solution of two sub-problems: an auction (matching game) *within* the coalition, and a coalition formation game to establish the coalitions [15].

Focusing on the latter problem, (16) can be seen either as an actual monetary loss, or as a virtual pricing imposed by a coordinator in order to control the energy losses: either way, it is assumed to be a *transferable* utility (TU). At any time, the agents in charge of the microgrids can inform each other of their energy needs (or surpluses). Then —through the evaluation of (16)— a group of agents may consider worthwhile to join and form a coalition $\mathcal{C}$. First, the involved agents must agree on the procedure to match sellers to buyers; then, the coalition's allocation, i.e., a mapping from the utility (16) to an allocation vector $\phi$ (each element $\phi_i$ is the payoff of agent $i$), has to be computed according to some specific fairness rules. Coalition will form if the payoff $\phi_i$ of at least one member increases without worsening the others' (and similarly for a coalition split). On the basis of this framework, the results presented in [50] show which coalitions would emerge in a typical network together with a discussion of the overhead due to coalition formation management.

Some open problems are remarked in [15]:

- Development of algorithms, based on auction theory or matching games, that can lead to optimal and stable associations between microgrids.
- Development of equilibrium concepts suitable to characterize the hybrid game resulting from a coalition formation game plus a matching game for sellers–buyers association.
- Formulation of utility functions able to capture, besides the power losses, the prices during energy trading and the costs of the required communications.

[32] Notice that the utility function in the form (16) does not include the dependence on the price resulting by the market closure. Nevertheless, it can be expanded to express that dependence.

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- Study of dynamic cooperative game models to model instantaneous variation in offer and demand characterizing renewable-based microgrids.
- Analysis of the impact of energy storage on the local energy trade.
- Practical implementation as a test bed.

### 5.2.2 Distributed control based on noncooperative games

The noncooperative game framework has been exploited in the literature for the simultaneous control of the generation and demand sides within a microgrid, in particular to model the competition arising among the users over the energy availability, and among the generators over the supply of the energy. A source–load balancing problem is analyzed as a game in [51] —on the basis of classical convex or mixed integer optimization— in order to study the resulting Nash equilibrium. For the single source / single load case, where the aim is to regulate the source's output voltage and the load's input power, two possible Nash equilibria exist: *(i)* the equilibrium in which both players achieve their goals and optimize their utilities, and *(ii)* an undesired equilibrium corresponding to the case of active constraints on the power transfer due to the line impedance, such that the load cannot meet its goal. Also, the use of Nash bargaining to improve a two-load game is discussed in [51]. According to [15] the future direction for research in this field should be:

- Study the impact of dynamics (e.g., variation in the power generation) on the outcome of the game.
- Develop algorithms to characterize the equilibria for multiplayer source–load games.

In [52] a different noncooperative framework is proposed to optimize the operation of a solar-powered microgrid with storage capabilities. Power produced or stored can be sold in a local market, besides being used locally. Demand and supply are modeled as a particular uncoordinated scenario, called the *Potluck Problem*. In the Potluck Problem, the agents do not communicate their needs; a nonrational behavior scheme is implemented instead to reach the desired equilibrium, which otherwise could not be reached with strictly rational behavior.

Saad et al. delineate the following future opportunities for the application of game theory to the smart grid framework:

- *Facility location games* can guide the definition of microgrids in the electricity network, as well as the *deployment of aggregation stations for electric vehicles.*
- Use of noncooperative games to model the switching of the microgrids' strategy between autonomous mode and operation along the main grid.
- Use Stackelberg games to study the coordination between microgrids and the main grid.

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- Study efficient coordination of information exchange among microgrids through *network formation games*.
- Model the dynamics of the interaction between electric vehicles and the grid using *dynamic game theory*.

## 5.3 Demand-side management in smart grids

Demand-side management consists in techniques for the manipulation of the energy consumption across the grid, by acting —directly or indirectly— on the users' demand. The implementation of such techniques is made possible by the communication devices embedded in the nodes of the smart grid, and is strongly motivated by the need of a more efficient use of the grid (e.g., peak shaving). Fundamental ways of manipulating the energy demand are the *direct load control*, where on the basis of a bilateral agreement the energy provider can act directly on the consumers' appliances, and *smart pricing*, where users are incentivized to shape their demand by means of real-time adjusted fares. The object of this techniques are primarily *shiftable* appliances, e.g., washing machines, dryers, dishwashers, or other manipulable loads such as air conditioning, recharging electric vehicles, etc.

Besides the straightforward benefits of such management for the energy providers and the entities in charge of the energy transmission infrastructure, advantages for consumers are foreseen too. Indeed, in most cases the costs for the producers are quadratically related to the demand they serve: this means that the cost of per-unit production of energy is heavily dependent on the point of operation of the plants. As the users are charged for these costs as well —and these costs depend on the global load served— *each user's objective function is coupled to the objective function of the rest of the users*. In other words, the minimization of the costs for the energy provider is expected to yield lower costs for the individual user.

A noncooperative formulation for such a case has been proposed in [53], based on a best response algorithm (it also ensures that no user has incentive to announce a fake demand schedule). The results show that the Nash equilibrium for the appliance scheduling game always exists, and all equilibria coincide with the strategy that also minimizes the costs for the energy provider. The advantages are most seen where each user possess a reasonable number of shiftable appliances. Notice however that when the priority for the user to switch a given appliance is accounted for in the objective function, the efficacy of the method is sensibly reduced.

### 5.3.1 Management of storage devices

Energy storage —for which plug-in electric vehicles will represent a fundamental resource— is envisioned as a key feature of the smart grid, yielding advantages such as integration of renewable sources, peak shaving, better use of the low-cost time slots of the energy market. Naturally this implies important challenges, related to the optimization of the pattern of use of storage devices (e.g., avoiding the con-

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current charge of several storage devices); indeed, an uncoordinated and concurrent use of such devices can easily lead to an increment of the market prices.

The development of suitable strategies is made possible by the communicational features offered by the smart grid. A deep study of the relationship between the storage patterns and the energy market trend, as well as the integration with renewable energy sources in order to broaden their employ, is of primary interest.

The study carried out in [54] explores the formulation of a noncooperative game for the optimal scheduling of a set of privately owned storage devices.[33] Results show that the Nash equilibria correspond to the storage patterns which minimize the *global generator costs*, given by

$$\sum_{h=1}^{H} \int_0^{q_h} b_h(q)dq$$

where $b_h(\cdot)$ is the supply curve and $q_h$ is the total energy traded by all users at time $h$. An adaptive best response scheme is proposed, based on a day-ahead prediction of the market, which is shown to converge to a Nash equilibrium [54].

According to Saad et al. [15], further investigation is required for the joint problem of both appliance and storage scheduling. For example, the inclusion of a player representing the energy provider, which may have interests that diverge from those of the consumers, may be the subject of future studies.

A fundamental line of research is the coordination of the demand among the users based on cooperative games. One of the goals could be an efficient load distribution, in order to better exploit the electrical infrastructure. The problem would be similar to that described in §5.2.1, but on a much more local level. However, privacy concerns have also to be accounted for in the demand-side cooperative management game. The intrinsic technical difficulties for estimating the exact strategies or objectives of other agents —limited information about other consumers is available at each user's level— can be dealt within the frameworks of games with incomplete and imperfect information (e.g., Bayesian games) as well as stochastic games [15].

[33] All the storage devices in the study are assumed to have the same characteristics.

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# 6 Conclusion

The present document collects fundamental notions of game theory reputed to be useful for the consolidation of the background knowledge required for the accomplishment of Task 3.1 and Task 3.2. Some of these notions, in particular those developed within the framework of noncooperative games (e.g., the Nash equilibrium) have been extensively applied in the distributed control literature. This does not hold for the analogous notions in cooperative games (e.g., the core), which in general are not tailored for dynamical environments. As a consequence, their application in control engineering is seldom encountered. Nevertheless, a growing number of recent pioneering works are contributing to the extension of cooperative game theory towards the control engineering field of application. The outcome of WP3 will expectedly serve to further bridge the gap between the game theory and the control engineering research fields.

The first part of this deliverable consists of a presentation of the core concepts of game theory. These concepts constitute a basis for the analysis of the interaction of control agents, as well as for the design of cooperative mechanisms for the management of complex systems. In the second part of the deliverable a brief literature survey is available. This survey is meant to trace an ideal path from the optimal model partitioning to the distributed control schemes whose principles support the idea of coalitional control.

In order to achieve the objectives of WP3, cooperative game theory is currently being employed to gain an insight into the relevance of the communicational links and the local controllers within a system of systems. In particular, the links of the network can be interpreted as players of a cooperative game whose characteristic function is given by a bound on the cost-to-go of the closed-loop system. It has been shown that proper allocation criteria for the game, e.g., the Shapley value, can provide information about the relevance of the links and agents involved in the distributed control problem. Likewise, if the objective function has an economical meaning, allocation criteria can be employed to distribute the resulting costs or benefits between the agents participating in the game.

Finally, this deliverable has been edited keeping in mind the development of solutions for the Dymasos case studies (particularly the HEP and AYESA cases). Future work is expected to involve the formulation of a demand behavior model, for the approximation of the variation in the demand induced by a pricing signal, and basic modeling of group dynamics, i.e., how users form coalitions and how these evolve according to a certain user welfare index.

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