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Abstract :

This deliverable presents two control schemes specifically developed within the coalitional framework foreseen as a part of the DyMaSoS project, to address the establishment of coalitions among the diverse agents involved in systems-of-systems. Global optimality concerns are tackled through a top-down approach, whereas individual interests are addressed with a novel bottom-up coalition formation process, based on the allocation provided by the Shapley value. This constitutes the preliminary result of the first two steps of Task 3.2, part of the WP3 *Coalitional Games in Systems of Systems*.
A graph-based interpretation of the autonomous coalition formation process is given as a result of WP2 *Economics-driven coordination and market-based management of systems of systems*.
Furthermore, a population-based setting, considered within Task 1.3 of WP1 *Population dynamics based approach to the management of systems of systems*, is presented for the study of mechanisms for the convergence to Nash equilibria of local cooperating agents.

Keywords :

Systems of systems, coalitional control, game theory, population-based control, distributed control, hierarchical control, system partitioning

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The DYMASOS Project

The well-being of the citizens in Europe depends on the reliable and efficient functioning of large interconnected systems, such as electric power systems, air traffic control, railway systems, large industrial production plants, etc. Such large systems consist of many interacting components. The sub-systems are usually managed locally and independently, according to different policies and priorities. The dynamic interaction of the locally managed components gives rise to complex behaviour and can lead to large-scale disruptions as e.g. black-outs in the electric grid.

Large interconnected systems with autonomously acting sub-units are called systems of systems. DYMASOS addresses systems of systems where the elements of the overall system are coupled by flows of physical quantities, e.g. electric power, steam or hot water, etc.

Within the project, new methods for the distributed management of large physically connected systems with local management and global coordination will be developed.

The DYMASOS Consortium consists of:

Participant no.	Participant organisation name	Participant short name	Country
1	Technische Universität Dortmund	TUDO	Germany
2	BASF SE	BASF	Germany
3	HEP-Operator distribucijskog sustava d.o.o	HEP	Croatia
4	INEOS Köln GmbH	INEOS	Germany
5	University of Seville	USE	Spain
6	University of Zagreb - Faculty of Electrical Engineering and Computing	UNIZG-FER	Croatia
7	ETH Zürich	ETH	Switzerland
8	RWTH Aachen University	RWTH	Germany
9	inno TSD	inno	France
10	Optimizacion Orientada a la Sostenibilidad SL	IDENER	Spain
11	euTeXoo GmbH	TEX	Germany
12	Ayesa Advanced Technologies SA	Ayesa AT	Spain

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Contents

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1 Executive summary

This deliverable presents the ongoing work carried on along the guidelines of Task 3.2 of WP3 *Coalitional Games in Systems of Systems*, concerning the development of techniques allowing to establish the formation of coalitions among the agents of a system of systems (SoS). Furthermore, the population-based coalitional approach developed in Task 1.3 of WP1 *Population dynamics based approach to the management of systems of systems* is presented.[1]

An aspect so far rarely contemplated in distributed control problems is the explicit consideration of individual (local) interests of the components of a complex system. Indeed, in order to allow fundamental properties of centralized control, such as system-wide optimality and stability, the majority of the literature about distributed control focuses on the overall system performance. However, when dealing with systems with a strong heterogeneous character, selfish interests may not be neglected. One of the critical points that have to be kept into consideration is the *diversity* characterizing the systems in object, yielding very complex interactions between the agents involved (see, e.g., the AYESA and HEP case studies contemplated within the DYMASOS project). In such settings, information about their relevance is of critical importance [1, 2]. Cooperative game theory is currently being employed to derive ad-hoc metrics [3, 4, 5].

In order to tackle such inherent aspects of SoS, the two basic architectures presented in this deliverable are contemplated within the DYMASOS project: *(i) top-down*, i.e., hierarchically supervised coalitional structure evolution, and *(ii) bottom-up*, i.e., autonomous negotiation between agents, leading to the emergence of cooperating clusters.

2 Introduction

Any system considered within the feedback control theory has a well-defined *global* organizational objective. Nevertheless, one must consider the possibility that certain components of the system may have their own interests [6]. The stronger the interaction among different parts of a system, the denser the communication required between the control agents (the extreme case being equivalent to centralized control), in order to compensate for incorrect models of the rest of the system [7]. In several cases the variables of a system can be grouped to highlight weakly coupled blocks, often revealing a natural topology. Within each block (commonly designated as *neighborhood*) dynamic interactions propagate quickly, affecting the rest of the system on a longer time scale [8].

Whenever possible it is desirable — for ease of implementation and reduction of the communication overhead — to formulate control laws based exclusively on local information [9, 10]. An interesting challenge is the online identification of subsystems' interactions [7], and the consequent adjustment of the control topology (thus varying its associated computational and communicational requirements). This is the rationale leading to *coalitional* control, where the strategy is adapted to the varying conditions

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of coupling between the agents, promoting the formation of coalitions among those most concerned [11].

Of great support to this line of research are the recent developments in data acquisition technologies (wireless networks, smart sensors) and in database management (cloud computing), which have yielded means of sharing measures and other information across a large-scale system in an efficient and flexible way [12]; also, thanks to the wide diffusion of smart mobile devices, the possibilities offered by human-in-the-loop control systems have been enhanced [13]. Several examples of the advances introduced by these new technologies on infrastructure systems can be identified, e.g., in modern traffic, water, and electricity networks [14, 15]. The improvement in the computational and communicational capabilities provided to local control devices constitutes now an additional impulse towards a new approach to distributed control problems: one where the cooperation between networked controllers is actively fostered and adapted in real-time to the state of the system.

Besides the smart grid (see [16] and references therein), and still strongly related to it, a further clear example of this trend is the great interest towards the utilization of Intelligent Transportation Systems (ITS). A consistent research effort is being recently devoted to this topic, typically involving different kind of game models in order to grasp the complex phenomena derived by the interaction of its heterogeneous user population. See, e.g., the analysis of the problem of choosing the EV charging station [15], the study of the consequences of a coalitional scenario among charging managers [17], or the setting for enabling Vehicle-to-Grid (V2G) operations through coalitions of users [18].

In the remainder of this document, two different approaches — hierarchically supervised and autonomous — to *coalitional control* are presented. More specifically, according to such control schemes, the structure of each agent's model predictive controller [19, 20] is adjusted following the time-variant coupling conditions. The first, discussed in Section 4, is a *top-down* approach, where the global coalitional structure is optimized at a supervisory layer. To address individual rationality instead, a *bottom-up* approach is proposed in Section 5: here the formation of coalitions is produced as the outcome of an autonomous bargaining procedure, following ad-hoc criteria — formulated on the basis of both cooperative control and game-theoretical fundamentals [7, 21].

Finally, Section 6 presents a population-based scenario where the agents can cooperate locally via a communication network.[1] Specifically, a quasi-aggregative game is considered, where the communications (and thus the coalitions) within the set of agents are expressed through a row stochastic matrix. Then, mechanisms for the convergence to a Nash equilibrium are studied.

[1]Work carried out by the Automatic Control Laboratory, ETH Zurich.

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3 Coalition formation

In general, coalition formation involves three steps. The first two are (i) *generation of the coalition structure and* (ii) *solution of the optimization problem for each coalition [22, 18]. Coalition formation is commonly studied in the form of characteristic function games, where a value $v(\mathcal{C})$ is associated to each possible coalition $\mathcal{C} \subseteq \mathcal{N}$, where $\mathcal{N}$ is the set of the agents. The value of a coalition structure $\mathcal{S}/\mathcal{C} \equiv \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{|\mathcal{S}/\mathcal{C}|}\}$ is thus obtained as the sum of the values of all coalitions constituting it, i.e.,*

$$V(\mathcal{S}/\mathcal{C}) = \sum_{\mathcal{C}\in\mathcal{S}/\mathcal{C}} v(\mathcal{C}).$$

Then, the optimal coalition structure is found as the one characterized by the highest value V^*. However, this problem has been demonstrated to be NP-complete [22]. To overcome this issue, several solution — resorting to heuristics, dynamic-programming, branch-and-bound algorithms, etc. — have been proposed in the literature (see [18, 3] and references therein).

One such solution — a hierarchical scheme that manipulates the controller topology with regard to both the current state of the system and the communication cost — is presented in Section 4. According to this scheme, the overall system is partitioned into coalitions working in a decentralized fashion, within which the agents are able to communicate with those other agents whose cooperation is most relevant.

The third and final step consists in the (iii) *distribution of the value of a coalition among its members. This last step, particularly interesting when local interests are of main importance in a given system, is tackled by means of the Shapley value through a different approach, based on autonomous bargaining, presented in Section 5.*

4 Supervised coalition structure generation

Often different parts of a networked large-scale system (think about infrastructures) are owned by independent entities, expectedly unwilling to coordinate their action unless strictly necessary. In addition, permanent communication between the various parts of the network can be impractical. Consequently, the use of a traditional centralized control approach is hampered, even when the whole system is owned and managed by a single entity. This motivates the two-layer hierarchical control scheme presented in this section. The main goal of the supervisory layer is to find the best compromise between control performance and communication costs by actively modifying the network topology. The actions taken at the supervisory layer alter the control agents' knowledge of the complete system, and the set of agents with which they can communicate. Each group of linked subsystems, or *coalition*, is independently controlled based on a decentralized model predictive control (MPC) scheme, managed at the bottom layer. As a consequence, those data links that do not yield a significant improvement of the control performance, compared with their relative cost of use, are disconnected. This feature is particularly interesting, e.g., for communication infrastructures based on battery-powered wireless devices [11].

The basic idea is to partition the centralized problem over a given number of local controllers or *agents*

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(see, e.g., [9]), designed to cooperate in order to satisfy certain global properties. Depending on the degree of dynamic interaction between the subsystems, the resulting control schemes are categorized in the literature either as distributed or as decentralized. In the first class the agents need to communicate to coordinate their operations [23, 24, 25, 26]. By contrast, in the second class the limited degree of interaction allows the agents to tackle their control tasks with no need of communication [27, 28]. Between these two classes lies the *coalitional* control, in which the topology of the controller is adapted to the varying coupling conditions between different parts of the system, promoting cooperation among the control agents most concerned at any given time. The formation of groups of cooperative agents based on the active coupling constraints is considered in [29]. The work of [30] describes a hierarchical framework where information among the agents is exchanged at each time step within clusters of strongly dynamically coupled subsystems, while a slower communication rate is required between different clusters. In [31], the complexity of the model predictive control problem of the Barcelona drinking water network is reduced by means of a partitioning algorithm, in order to control in a hierarchical-distributed manner the resulting subnetworks. In [32] a flexible hierarchical MPC scheme is proposed for a hydro-power valley, where the priority of the agents in optimizing their control actions can be rearranged according to the different operational conditions.

A multi-agent control scheme based on the same basic idea is discussed in [3], where a design method that guarantees closed-loop stability for the proposed scheme is provided. The network topology optimization problem is posed as a cooperative game, in order to study the relevance of the different links and agents under a game-theoretical perspective (in view, e.g., of fault-tolerant policies). From this perspective, each network topology used is interpreted as a coalition of links that evolves in order to optimize the expected evolution of the closed-loop system.

Analogously, the present scheme focuses on how the interaction between the subsystems varies with time [11]. A cost on the coordination effort is considered, so that the overall controller architecture evolves by trading off control performance for savings on communication costs. Such costs are accounted for by means of ad-hoc indices related to the number of data links enabled in order to establish communication between every member of the coalition. Further criteria may be employed to evaluate cooperation costs, based on, e.g., the total number of agents involved in the coalition, the number of decision variables and/or constraints of the aggregate problem, reflecting the computational requirements [33]. As a result, coordination between agents is promoted whenever the dynamic interaction between their corresponding subsystems is critical.

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4.1 Problem statement

The dynamics of any subsystem $i \in \mathcal{S} = \{1, \ldots, N\}$ are described by the linear state-space model:

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + w_i(k), \tag{1a}$$

$$w_i(k) = \sum_{j \in \mathcal{N}_i} A_{ij}x_j(k) + B_{ij}u_j(k), \tag{1b}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{m_i}$ are the state and input vectors respectively, and $w_i \in \mathbb{R}^{r_i}$ describes the influence on x_i of the neighbors' states and inputs. In (1b), $x_j \in \mathbb{R}^{n_j}$ and $u_j \in \mathbb{R}^{m_j}$ are the state and input vectors of each neighbor $j \in \mathcal{N}_i$ of subsystem i. The neighborhood set $\mathcal{N}_i$ is defined as:

$$\mathcal{N}_i = \{j \in \mathcal{S} | A_{ij} \neq \mathbf{0} \vee B_{ij} \neq \mathbf{0}, j \neq i\}, \tag{2}$$

i.e., it contains any subsystem $j \neq i$ whose state and/or input produce some effect on the dynamics of subsystem i.

4.2 Exchange of information

All the control agents can communicate through a data network whose topology is described by means of the undirected graph $\mathcal{G} = (\mathcal{S}, \Lambda)$, where to each subsystem in $\mathcal{S}$ is assigned a node. Let $\mathcal{L} \subseteq \mathcal{S} \times \mathcal{S}$ be the set of edges corresponding to the existing communication links between the agents. Each link $\ell_{ij} = \{i, j\} = \{j, i\} = \ell_{ji} \in \mathcal{L}$ can be either enabled or disabled. Then the *network topology* $\Lambda(k) \subseteq \mathcal{L}$ is defined as the set of links enabled at a given time, i.e., $\ell_{ij} \in \Lambda(k)$ if and only if it is enabled at time k. Each active link has a cost $c_\ell > 0$ per time of use. This cost can vary, e.g., as a function of the available bandwidth.

Definition 1 *Any two agents are said to be* connected *if and only if there exists a path between them in* $\mathcal{G} = (\mathcal{S}, \Lambda)$.

Assumption 2 *Any two agents can communicate if and only if they are connected.*

From Definition 1 and Assumption 2 it follows that any given network topology induces a partition of the whole agent set $\mathcal{S}$ into disjoint communication components [34]. As agents within the same communication component will benefit from cooperation — i.e., sharing information in order to aggregate their control tasks — we will refer to such components as *coalitions*, and the partition resulting from a given network topology $\Lambda(k)$ will be denoted as $\mathcal{S}/\Lambda = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{|\mathcal{S}/\Lambda|}\}$, where $|\cdot|$ represents the cardinality of the set. To ease the notation, let us define the set of indices $\mathcal{S}_\Lambda = \{1, \ldots, |\mathcal{S}/\Lambda|\}$. Any partition $\mathcal{S}/\Lambda$ originates a set of coalitions satisfying the following conditions [35]:

(i) $\mathcal{C}_i \neq \varnothing$, $\forall i \in \mathcal{S}_\Lambda$;

(ii) $\bigcup_{i=1}^{|\mathcal{S}/\Lambda|} \mathcal{C}_i = \mathcal{S}$;

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(iii) $\mathcal{C}_i \cap \mathcal{C}_j = \varnothing$, $\forall i, j \in \mathcal{S}_\Lambda$, $i \neq j$.

The number of coalitions $|\mathcal{S}/\Lambda|$ associated with any topology Λ pertains to the interval $[1, N]$, whose extremes represent the centralized control case (all the N subsystems connected[2]) and the case where each subsystem forms a coalition on its own (all links disabled).

4.3 Coalition dynamics

Thus, to describe the dynamics of each coalition $\mathcal{C}_i \in \mathcal{S}/\Lambda$, the following extension of (1a) holds:

$$\xi_i(k+1) = \Xi_{ii}\xi_i(k) + \Upsilon_{ii}\upsilon_i(k) + \omega_i(k), \tag{3}$$

where ξ_i and υ_i are respectively the state and input vectors of coalition $\mathcal{C}_i$, composed by stacking the vectors of all the subsystems in the coalition:

$$\xi_i = \{x_s\}_{s\in\mathcal{C}_i}, \quad \upsilon_i = \{u_s\}_{s\in\mathcal{C}_i}, \quad i \in \mathcal{S}_\Lambda.$$

As an extension of (1b), ω_i expresses the influence of the neighbor coalitions' states and inputs on ξ_i:

$$\omega_i(k) = \sum_{j\in\mathcal{N}_{\mathcal{C}_i}} \Xi_{ij}\xi_j(k) + \Upsilon_{ij}\upsilon_j(k), \tag{4}$$

where the set of neighbors $\mathcal{N}_{\mathcal{C}_i}$ is an extension of (2), indexing any coalition $\mathcal{C}_j$, $j \neq i$ whose state and/or inputs produce some effect on the dynamics of (any subsystem inside) coalition $\mathcal{C}_i$. Matrices Ξ_{ii}, Ξ_{ij}, Υ_{ii} and Υ_{ij} are composed accordingly.

4.4 Control objective

In the application considered here, the control objective is to regulate the state of all subsystems while minimizing a cost that depends on the state and input trajectories. An additional term in the cost function will take into account the use of network resources.

Denoting the shifted state and input of coalition $\mathcal{C}_i$ w.r.t. their setpoint $\bar{\xi}_i$ and $\bar{\upsilon}_i$, as $\zeta_i = \xi_i - \bar{\xi}_i$ and $\nu_i = \upsilon_i - \bar{\upsilon}_i$, respectively, the cost function can be divided into a term J_s representing the optimal performance objective, and a term J_n expressing the network-related cost:

$$J_{s,i} = \sum_{t=0}^{N_p-1} \left(\zeta_i^\intercal(t|k)Q_i\zeta_i(t|k) + \nu_i^\intercal(t|k)R_i\nu_i(t|k)\right) + \zeta_i^\intercal(N_p|k)P_i\zeta_i(N_p|k), \tag{5a}$$

$$J_{n,j} = N_p\frac{c_\ell}{2}n_{\ell,j}(\Lambda), \tag{5b}$$

where the notation $x(t|k)$ corresponds to the value of x predicted at time $k+t$, based on the knowledge at time k, and $Q_i \geq 0$, $R_i > 0$ and $P_i = P_i^\intercal > 0$ are constant weighting matrices. In (5b), $n_{\ell,j}(\Lambda)$ is the number of active links *directly* connecting agent j to other agents according to the network topology Λ.

[2]Notice that, according to Assumption 2, this condition does not necessarily require all the links to be active.

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Note that each agent shares the cost of a link with the agent located at the other side of that link. The overall control problem can be posed as the following receding-horizon optimization:

$$\min_{\boldsymbol{\upsilon},\Lambda} \sum_{i\in\mathcal{S}_\Lambda} J_{s,i}(\xi_i(k),\upsilon_i,\Lambda)+\sum_{j\in\mathcal{S}} J_{n,j}(\Lambda) \tag{6}$$

s.t.

$$\begin{aligned}\xi_i(t+1|k) &= \Xi_{ii}\xi_i(t|k)+\Upsilon_{ii}\upsilon_i(t|k)+\hat{\omega}_i(k)\\ \xi_i(t|k) &\in \mathcal{X}_i,\ \forall t\in[0,N_p]\\ \upsilon_i(t|k) &\in \mathcal{U}_i,\ \forall t\in[0,N_p-1]\\ \xi_i(0|k) &= \xi_i(k)\\ \Lambda &\subseteq \mathcal{L},\end{aligned}$$

where $\mathcal{X}_i$ and $\mathcal{U}_i$ respectively denote the state and input constraint sets for the subsystems in coalition $\mathcal{C}_i$. Notice that (4) cannot be used directly, as coalition $\mathcal{C}_i$ has no knowledge of the states and inputs of external subsystems. An estimate $\hat{\omega}_i$ of the perturbation they cause on ξ_i is thus computed at each time step k, and its value is assumed constant along the prediction horizon. Problem (6) constitutes a dynamic programming optimization with mixed integer variables, which is generally not practical to solve. Since any topology corresponds to a partition of the global system, the composition of the resulting coalitions' state and input vectors and matrices will implicitly depend on Λ. The choice of the network topology is made within a discrete set whose size, in the general case, grows exponentially with the number of links. In the remainder, we formulate a hierarchical multi-agent control algorithm which provides a suboptimal, yet less computationally expensive solution.

4.5 The control algorithm

The control architecture resulting from the proposed approximation of (6) is organized on two layers: the top layer will be in charge of the choice of the network topology, whereas the bottom layer will handle — separately for each coalition — the eventual estimation of the dynamic influence of external subsystems as well as the real-time control tasks. At the bottom layer, the control is decentralized into the coalitions arising from the partition $\mathcal{S}/\Lambda^*$. With the term *decentralized* we designate the complete absence of communication among different coalitions; agents within a coalition share their information at each sample time. As a consequence, the term ω_i that models the effect of neighboring coalitions cannot be computed through (4), and each coalition needs to employ an estimate $\hat{\omega}_i$. Issues related with such estimation are strongly case-related, and out of the scope of this document. In general, it is desirable that inter-coalition coupling — when not sufficiently weak to be negligible — shows relatively slow dynamics: this allows it to be reasonably approximated as a constant perturbation. Whenever this cannot be the case, the employ of robust or stochastic approaches may be needed. Next, further details are given about the operation of the top layer.

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The discrete-valued part of (6), constituting the most computationally demanding part of the problem, is handled at the top layer. For this reason, its solution is computed on a coarser time scale (w.r.t. the sample time required for the control of the system), and the corresponding topology maintained during the following interval T_Λ. In order to select the most appropriate global control structure, several network topologies are compared. Let $\mathfrak{L}^+ = \{\Lambda_1, \Lambda_2, \ldots, \Lambda_{|\mathfrak{L}^+|}\}$ be the set of possible network topologies to be evaluated. Then, let us define the function $J : \mathbb{R}^n \times \mathcal{L} \mapsto \mathbb{R}$ as follows [36]:

$$J(\boldsymbol{\xi}, \Lambda) = \sum_{i \in \mathcal{S}_\Lambda} \zeta_i^\mathsf{T} P_i \zeta_i + c_\ell |\Lambda| T_\Lambda, \;\; \Lambda \in \mathfrak{L}^+, \tag{8}$$

where $P_i = P_i^\mathsf{T} > 0$, $|\Lambda|$ is the number of enabled links and c_ℓ is the cost of use of one link, considered over the interval T_Λ. It is not pragmatic to see $\mathfrak{L}^+$ as the set containing every possible configuration of links. Because the number of all possible topologies grows exponentially with the number of links, the set $\mathfrak{L}^+$ should be defined as a reasonably sized set of *relevant* topologies for the system to be controlled. The composition of $\mathfrak{L}^+$ could either be static or evolving in relation with, e.g., the current state of the system, the network constraints, or the willingness to cooperate among the agents. Of all the configurations considered at a given moment, the one giving the optimal value of (8), denoted as $\Lambda^* \in \mathfrak{L}^+$, will be applied during the next interval T_Λ.

As a consequence to the choice of any given topology Λ, the set of agents is partitioned into a specific set of coalitions $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{|\mathcal{S}/\Lambda|}\}$. To attain the optimal performance objective (5a), a feedback gain $\mathbb{K}$ for the whole system is computed at the top layer. Notice that constraints are accounted for in the bottom layer: as a consequence, the feedback gain $\mathbb{K}$ computed at the top layer is relative to the unconstrained problem.[3] In conformity with the system partition $\mathcal{S}/\Lambda$, $\mathbb{K}$ will be composed of a set of decentralized feedback gains, each one associated to a coalition, i.e., $\mathbb{K} = diag\{K_1, \ldots, K_{|\mathcal{S}/\Lambda|}\}$. Let $\mathbb{P} > 0$ be the block matrix having $\{P_1, \ldots, P_{|\mathcal{S}/\Lambda|}\}$ on its diagonal, and consider the Lyapunov function $V(\boldsymbol{\xi}) = \boldsymbol{\xi}^\mathsf{T}\mathbb{P}\boldsymbol{\xi}$, where $\boldsymbol{\xi} \triangleq \{\xi_i\}_{i\in\mathcal{S}_\Lambda}$ is the global state vector, permuted according to the partition $\mathcal{S}/\Lambda$. For $V(\boldsymbol{\xi})$ to constitute an upper bound on the infinite-horizon performance objective, the constraints of the following LMI problem have to be satisfied (see, e.g., [37]):

$$\max_{\mathbb{K},\mathbb{P}} \mathbf{Tr}\mathbb{P}^{-1} \tag{9}$$

s.t.

$$\mathbb{P} = \mathbb{P}^\mathsf{T} > 0,$$
$$(\boldsymbol{\Xi} + \boldsymbol{\Upsilon}\mathbb{K})^\mathsf{T}\mathbb{P}(\boldsymbol{\Xi} + \boldsymbol{\Upsilon}\mathbb{K}) - \mathbb{P} \leq -\mathbf{Q} - \mathbb{K}^\mathsf{T}\mathbf{R}\mathbb{K},$$

where $\boldsymbol{\Xi}$ and $\boldsymbol{\Upsilon}$ are respectively the global state and input matrices, composed to match $\boldsymbol{\xi}$ and $\boldsymbol{\upsilon} \triangleq \{\upsilon_i\}_{i\in\mathcal{S}_\Lambda}$. Similarly, $\mathbf{Q} = diag\{Q_1, \ldots, Q_{|\mathcal{S}/\Lambda^*|}\} \geq 0$ and $\mathbf{R} = diag\{R_1, \ldots, R_{|\mathcal{S}/\Lambda^*|}\} > 0$ are the global weighting matrices.

[3] Indeed, for the grand coalition, the feedback law $\mathbb{K}$ will coincide with the LQR gain.

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By the solution of (9), a set of feedback control laws $\upsilon_i = K_i \xi_i$ which minimize $V(\boldsymbol{\xi})$ and a set of matrices P_i, $i \in \mathcal{S}_\Lambda$ is obtained. These matrices are then used to compute the value of (8) and find its minimizer Λ^*.

Remark 3 *Notice that the evaluation of different network topologies is independent and can be executed in parallel on a multi-processor platform. Also, the set of control laws associated with any network topology could be stored and reused whenever the same topology is considered again, without the need of solving more than once the relative LMI problem.*

For further details the reader is referred to [11, 3].

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5 Autonomous coalition formation

In order to allow fundamental properties of centralized control, such as optimality and stability, the individual (local) interests of the components of a large-scale system are typically subordinated for the overall system performance. This occurs as well in the supervised coalition structure generation architecture presented in Section 4, where a tradeoff between global optimality and inter-agent information exchange is followed. However, when dealing with systems characterized by a strong heterogeneous character, selfish interests may not be neglected. To address this issue, a bottom-up approach to coalitional control is proposed, where the structure of each agent's model predictive controller is adapted to the time-variant coupling conditions of the system, promoting the formation of coalitions as outcome of a pairwise bargaining procedure.

5.1 Problem statement

Consider a set $\mathcal{S} = \{1, \ldots, N\}$ of discrete-time linear processes, coupled through the inputs, such that each can be modeled by the following state-space equation:

$$x_i(k+1) = A_i x_i(k) + \sum_{j \in \mathcal{N}_i} B_{ij} u_{ij}(k) + \sum_{j \in \mathcal{N}_i} B_{ji} u_{ji}(k). \tag{10}$$

In (10), $x_i \in \mathbb{R}^n_i$ is the state of the ith subsystem, and $u_{ij} \in \mathbb{R}^m_{ij}$ is the control action applied by subsystem j on subsystem i (and viceversa for u_{ji}). $\mathcal{N}_i \subseteq \mathcal{S} \setminus \{i\}$ is the set of indices identifying the subsystems whose inputs produce a direct effect on subsystem i. Without loss of generality, this coupling can be assumed to be symmetrical, such that for all $j \in \mathcal{N}_i$, we have $i \in \mathcal{N}_j$ as well. $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_{ij} \in \mathbb{R}^{n_i \times m_{ij}}$ and $B_{ji} \in \mathbb{R}^{n_i \times m_{ji}}$ are the state transition matrices.

Real large-scale systems that show analogies (about given operating conditions) with the above formulation are, e.g., drinking water networks composed by interconnected water tanks [31], irrigation canals, modeled by integrator-delay models in [38, 11, 39], supply chains [40], traffic networks and power grids [41].

The performance of each subsystem $i \in \mathcal{S}$ is measured through a local stage cost defined as:

$$\ell_i(k) = (x_i(k) - \bar{x}_i)^\intercal Q_i (x_i(k) - \bar{x}_i) + (u_i(k) - \bar{u}_i)^\intercal R_i (u_i(k) - \bar{u}_i), \tag{11}$$

where x_i and $u_i \triangleq \{u_{ji}\}_{j \in \mathcal{N}_i}$ are respectively the state and input vectors of subsystem i. The matrices Q_i and R_i weigh the deviation of state and input from their reference.

A central point in this discussion is the assumption that the agents — each of which is assigned the task of controlling one subsystem — act in order to minimize their stage cost (11). More specifically, they are assumed to be *rational* and *selfish*. In the scenario considered here, a decentralized architecture results, where each subsystem is governed by a local model predictive (MPC) controller. The input actions to the process are obtained as the solution of the following optimization problem:

$$\min_{\mathbf{u}_i} \sum_{t=k}^{k+N_p} \hat{\ell}_i(t) \tag{12}$$

s.t.

$$\hat{x}_i(t+1) = A_i\hat{x}_i(t) + \sum_{j\in\mathcal{N}_i} B_{ji}u_{ji}(t) + \sum_{j\in\mathcal{N}_i} B_{ij}\hat{u}_{ij}(t), \tag{13a}$$

$$u_i(t) \in \mathcal{U}_i, \quad \forall t \in \{k,\ldots,k+N_p-1\}, \tag{13b}$$

and $\hat{x}_i(0) = x_i(k)$. The optimization variable

$$\mathbf{u}_i \triangleq [u_i(k),\ldots,u_i(k+N_p-1)]$$

is a column vector composed of the sequence of control actions along the prediction horizon of length N_p. At time k the first element $\mathbf{u}_i^*(0) \triangleq u_i(k)$ of the minimizing sequence is applied to the system, and the problem is solved again at subsequent time instants in a receding horizon fashion [20].

In case no information is exchanged among the agents, problem (12) has to be solved over an estimated stage cost $\hat{\ell}_i$, derived using (13a) on (11). The inputs of neighboring agents are unknown and need to be estimated: the specific solution of the estimation problem is out of the scope of this document; some related remarks can be found in Section 4.5. By means of the autonomous coalition generation framework considered in the remainder, agents will be able to expand their knowledge of the rest of the system and to jointly agree on the value assigned to the inputs.

5.2 Autonomous coalition structure generation

In the remainder, the term *player* may refer either to a single control agent or to a group of agents that, as a consequence of their participation into the same coalition, act as a single entity. In order to keep the notation simple, indices $\{1, 2\}$ will be used to designate the players; furthermore, the notation $\{1\cup 2\}$ will refer to their merger. The subsystems involved in either part of a given bargaining process are identified by the sets $\mathcal{P}_1 \subset \mathcal{S}$ and $\mathcal{P}_2 \subset \mathcal{S}$: the coalitions of agents corresponding to these sets constitute the two players. Thus, the set of subsystems forming the merger will be designated as $\mathcal{P}_{1\cup 2} \triangleq \mathcal{P}_1 \cup \mathcal{P}_2$. Overlapping coalitions are not considered, so $\mathcal{P}_1 \cap \mathcal{P}_2 = \varnothing$.

The states and inputs of every subsystem constituting the coalitions that take part in the bargaining process are gathered into the player's state and input vectors, defined as:

$$\xi_1 \triangleq \{x_i\}_{i\in\mathcal{P}_1}, \qquad \upsilon_1 \triangleq \{u_i\}_{i\in\mathcal{P}_1},$$

$$\xi_2 \triangleq \{x_j\}_{j\in\mathcal{P}_2}, \qquad \upsilon_2 \triangleq \{u_j\}_{j\in\mathcal{P}_2},$$

where $\mathcal{P}_1 \subset \mathcal{S}$, $\mathcal{P}_2 \subset \mathcal{S}$, and $\mathcal{P}_1 \cap \mathcal{P}_2 = \varnothing$. Finally, the merger state and input vectors are composed according to

$$\xi_{12}^\intercal = [\xi_1^\intercal\ \xi_2^\intercal], \quad \upsilon_{12}^\intercal = [\upsilon_1^\intercal\ \upsilon_2^\intercal].$$

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The objective now is to establish a possible criterion for endogenous coalition formation, *oriented at networks of coupled dynamical systems*, guaranteeing a fair distribution of benefits. Starting from the performance improvement offered by cooperative control, regarded as coalitional benefit, we point out the issues — from the individual control agent standpoint — of the absence of redistribution of such benefit. Then, we propose a possible solution based on the Shapley value [42].

5.2.1 Joint benefit through cooperation

The bargaining contemplated here is based on an index accounting for both control performance and cooperation-related costs:

$$J_i = \sum_{t=k}^{k+N_p} \ell_i(t) + \chi_i, \quad i \in \{1, 2, 1{\cup}2\}, \tag{14}$$

where ℓ_i is the quadratic stage cost (evaluated over the horizon N_p) associated with either the merger or the individual players,

$$\ell_i = (\xi_i(k) - \bar{\xi}_i)^\intercal \mathbf{Q}_i (\xi_i(k) - \bar{\xi}_i) + (\upsilon_i(k) - \bar{\upsilon}_i)^\intercal \mathbf{R}_i (\upsilon_i(k) - \bar{\upsilon}_i), \tag{15}$$

where $\mathbf{Q}_i = \mathrm{diag}(\{Q_j\}_{j\in\mathcal{P}_i})$, $\mathbf{R}_i = \mathrm{diag}(\{R_j\}_{j\in\mathcal{P}_i})$ for $i \in \{1, 2\}$, whereas for the merger

$$\mathbf{Q}_{1\cup2} = \begin{bmatrix} \mathbf{Q}_1 & 0 \\ 0 & \mathbf{Q}_2 \end{bmatrix}, \quad \mathbf{R}_{1\cup2} = \begin{bmatrix} \mathbf{R}_1 & 0 \\ 0 & \mathbf{R}_2 \end{bmatrix}.$$

The cooperation costs are expressed in (14) by the term χ_i; notice that for $i \in \{1, 2\}$ these costs involve only player i internal communication.

Remark 4 *As pointed out in Section 4, costs required for the cooperation to take place can be accounted for by means of ad-hoc indices related, in this case, with the total number of agents involved in the bargaining process, or with the number of data links enabled in order to establish communication between every member of the coalition. Different measures may be employed to evaluate cooperation costs, based on, e.g., the number of decision variables and/or constraints of the aggregate problem, reflecting the computational requirements.*

To predict the result of either agreement or disagreement on the formation of the new coalition, MPC problem (16) is solved for the three possible cost functions:

$$\min_{\boldsymbol{\upsilon}_i} \sum_{t=k}^{k+N_p} \hat{\ell}_i(t, \upsilon_i(t)), \; \forall i \in \{1, 2, 1{\cup}2\}, \tag{16}$$

where $\hat{\ell}_i$ is the estimated stage cost (see Section 5.1), based on the knowledge available within either set of agents $\mathcal{P}_i$ if $i \in \{1, 2\}$ or, in case of merger, the joint knowledge provided by $\mathcal{P}_1 \cup \mathcal{P}_2$. The constraints for (16) are:

$$\xi_i(t+1) = \mathbb{A}_i \xi_i(t) + \mathbb{B}_i \upsilon_i(t) + \mathbb{E}_i \hat{\upsilon}_{-i}(t) \tag{17a}$$

$$\upsilon_i(t) \in \bigotimes_{j\in\mathcal{P}_i} \mathcal{U}_j, \; i \in \{1, 2, 1{\cup}2\}, \tag{17b}$$

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for all $t \in \{k, \ldots, k+N_p-1\}$, where $\hat{\upsilon}_{-i} \triangleq \{\hat{u}_j\}_{j\in\mathcal{M}_{-i}}$ gathers the estimated effect of the agents not involved in the optimization (more details in (18)). The state transition matrices for the individual players are defined as $\mathbb{A}_1 = \text{diag}(\{A_i\}_{i\in\mathcal{P}_1})$ and $\mathbb{B}_1 = [\mathbf{B}_{ij}]$ where, for all $i \in \mathcal{P}_1$, $\mathbf{B}_{ii}$ is such that

$$\mathbf{B}_{ii}u_i = \sum_{j\in\mathcal{N}_i} B_{ji}u_{ji},$$

with $u_i = \{u_{ji}\}_{j\in\mathcal{N}_i}$ (as defined in Section 5.1), and

$$\mathbf{B}_{ij}u_j = B_{ij}u_{ij}, \quad \forall j \in \mathcal{P}_1 \setminus \{i\},$$

with $u_j \triangleq \{u_{lj}\}_{l\in\mathcal{N}_j}$. Notice that $B_{ij} \neq \mathbf{0}$ if and only if $j \in \mathcal{P}_1 \cap \mathcal{N}_i$. Matrices for player 2 are composed likewise. For the case $i = 1\cup 2$, the state transition matrices in (17a) are

$$\mathbb{A}_{1\cup 2} = \begin{bmatrix} \mathbb{A}_1 & 0 \\ 0 & \mathbb{A}_2 \end{bmatrix}, \quad \mathbb{B}_{1\cup 2} = \begin{bmatrix} \mathbb{B}_1 & \Upsilon_{12} \\ \Upsilon_{21} & \mathbb{B}_2 \end{bmatrix},$$

where $\Upsilon_{12} = [\mathbf{B}_{ij}]$, and for all $i \in \mathcal{P}_1$

$$\mathbf{B}_{ij}u_j = B_{ij}u_{ij}, \quad \forall j \in \mathcal{P}_2,$$

with $u_j \triangleq \{u_{lj}\}_{l\in\mathcal{N}_j}$. Notice that $B_{ij} \neq \mathbf{0}$ if and only if $j \in \mathcal{P}_2 \cap \mathcal{N}_i$. An analogous definition holds for Υ_{21}. The last term in (17a) accounts for the (estimated) effect of the agents not involved in the optimization, i.e., every $j \in \mathcal{M}_{-i}$ where

$$\mathcal{M}_{-i} = \{j|\, j \in \mathcal{M} \setminus \mathcal{P}_i\}, \tag{18a}$$

$$\mathcal{M} = \bigcup_{l\in\mathcal{P}_i} \mathcal{N}_l, \tag{18b}$$

for each $i \in \{1, 2, 1\cup 2\}$. So $\mathbb{E}_1 = [\mathbf{E}_{ij}]$ where, for all $i \in \mathcal{P}_1$

$$\mathbf{E}_{ij}\hat{u}_j = B_{ij}\hat{u}_{ij}, \quad \forall j \in \mathcal{M}_{-i},$$

with $\hat{u}_j \triangleq \{\hat{u}_{lj}\}_{j\in\mathcal{N}_j}$. An analogous definition holds for player 2.

Finally, necessary condition for the coalition $\mathcal{P}_1 \cup \mathcal{P}_2$ to form is:

$$J^*_{1\cup 2} \leq J^*_1 + J^*_2, \tag{19}$$

where the superscript '$*$' designates the values of (14) corresponding to the minimizing input sequence obtained by the solution of (16). Notice that (19) constitutes the foundation of distributed cooperative MPC algorithms (see, e.g., [43]), where local agents optimize an index that reflects the global plant performance.

Remark 5 *Any new coalition will be product of the union of two players,* and thus of all agents they involve. *The present approach is based on the performance of the player as a whole and not on that of its individual components. One basic advantage of such approximation is that of avoiding the combinatorial explosion of the possible configurations that would arise otherwise.*

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5.2.2 Individual rationality

The approximation specified in Remark 5 is tantamount to assuming that an agreement that is beneficial for the entire coalition will be beneficial for each of its members too. Since the premise here is that the point of view of each single agent is based on its individual cost (11) (agents are rational and selfish), it is essential to check this condition. However, here we test this condition on the coarsest scale, i.e., over each pair of bargaining players. Let us first define

$$J_{1\cup 2}^{(j)} \triangleq \sum_{t=k}^{k+N_p} \ell_{1\cup 2}^{(j)}(t) + \chi_{1\cup 2}^{(j)}, \quad j = \{1,2\}, \tag{20}$$

as the contribution of player j to the merger cost $J_{1\cup 2}$. The stage cost employed in (20) is the component of (15) relative to one of the players, for the case in which the merger is produced (i.e., for the state and input trajectories obtained by solving the MPC problem (16) for $i = 1\cup 2$). The cooperation cost index $\chi_{1\cup 2}^{(j)}$ is a proper share of the cooperation costs.

It can be verified (by assuming, for simplicity, no costs for communications) that

$$J_{1\cup 2} \leq J_1 + J_2 \;\not\Rightarrow\; J_{1\cup 2}^{(j)} \leq J_j, \quad \forall j \in \{1,2\}. \tag{21}$$

Indeed, the verification of (19) does not guarantee *individual* benefit to both players — unless some means of transferring the value between them is provided. On the grounds of individual rationality, a new coalition is formed if and only if a secure benefit can be granted to their future members.[4] In particular, either player will accept participating in the merger $\mathcal{P}_1 \cup \mathcal{P}_2$ whenever the following individual rationality requirement is fulfilled:

$$J_{1\cup 2}^{(j)} \leq J_j, \quad \forall j \in \{1,2\}. \tag{22}$$

5.2.3 Coalitional TU algorithm

Algorithms based on the sole improvement of the joint benefit, or on individual rationality concerns, are indeed best motivated when the enhancement of performance achieved through the coalition cannot be translated into economical units and, most importantly, cannot be transferred (as recompense) from one player to the other. In the remainder we assume instead that the index (14) can be economically measured. Then, we consider the possibilities opened whenever a value equivalent to the benefit achieved through the merger, i.e.,

$$\Pi = J_1 + J_2 - J_{1\cup 2}, \tag{23}$$

can be transferred between the players; in other words, we consider a transferable utility (TU) scenario [44]. Notice that in such a scenario it is possible to relax condition (22), since it can be fulfilled by means of a proper *a posteriori* redistribution of the utility among the players.

[4]Individual rationality requires that the payoff obtained through the merger has to be equal, if not better, than the one obtained through a solitary strategy.

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Now we aim at distributing the entire profit (or, equivalently, to reallocate the control cost) of the merger such that both players are satisfied. Since a TU game contemplates the division of the total benefit among the players, if rational players come to an agreement, they will agree on achieving the largest possible payoff. Such joint agreement is referred to as *cooperative strategy*. Note that a similar agreement will belong to the Pareto front (no allocation can make a player better off without making the other player worse off). The payoffs achieved with solitary strategies constitute the *disagreement point*, i.e., (J_1, J_2): a player would not accept a payoff smaller than its own disagreement point.

We see that the Shapley value, due to its inherent properties [42], appears as a clear candidate to compute such allocation. In particular, the most relevant to our scope is the *carrier* axiom [21]: it implies that the benefit is allocated among the players actually contributing to the performance improvement of the merger. More formally, any coalition $\mathcal{C}$ for which it holds that $v(\mathcal{C}') = v(\mathcal{C}' \cap \mathcal{C})$, for any other coalition $\mathcal{C}'$ in the set of players, is referred to as the *carrier* of the game. According to the carrier axiom, the value of the game is confined within the carrier coalition. Notice that this implies *(i) efficiency* of the allocation, i.e., the sum of the payoffs assigned to each player in the game equals the aggregate benefit, and *(ii) dummy players* receive a null payoff, i.e., the benefit is distributed only among the players contributing to the value of the coalition.[5] Moreover, individual rationality is guaranteed by the *superadditivity* condition described by (19). Hence, the allocation provided by the Shapley value qualifies as an *imputation* (i.e., an allocation which is both efficient and individually rational). Notice that for two-player TU games any imputation is in the *core*. This means that — in the bargaining scenario analyzed in this work — any imputation given by the Shapley value will provide stability to the merger.

The Shapley value for a two-player TU game can be explicitly written as:

$$\phi_i = \frac{1}{2} v(\mathcal{P}_i) + \frac{1}{2} \left[v(\mathcal{P}_1 \cup \mathcal{P}_2) - v(\mathcal{P}_j) \right], \tag{24}$$

for each player $i \in \{1, 2\}$, and $j \in \{1, 2\} \setminus \{i\}$. In order to obtain a measure of a fair individual cost, inversely related to the *importance* of either player in the merger, we start by defining the coalition values to be employed in (24) as in (25).

$$\begin{aligned} v(\mathcal{P}_1) &= J_1, \\ v(\mathcal{P}_2) &= J_2, \\ v(\mathcal{P}_1 \cup \mathcal{P}_2) &= J_{1\cup 2}. \end{aligned} \tag{25}$$

Following the definitions in (25), the payoffs computed through (24) take the form

$$\phi_i = \frac{1}{2} J_i + \frac{1}{2} (J_{1\cup 2} - J_j), \quad j \in \{1, 2\} \setminus \{i\}, \tag{26}$$

expressing the quota relative to each player in the compound cost index $J_{1\cup 2}$, according to the intrinsic fairness of the Shapley value.

[5] In our scenario, dummy players will be those who do not show any dynamic coupling with the (agents within the) other player).

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Provided that the necessary condition for coalition formation (19) is fulfilled, consider the general case in which the cost associated to the unilateral strategy of one of the players (designated from now on as player 1) shows a greater associated cost (w.r.t. the other player), i.e.,

$$J_{1\cup 2} \leq J_1 + J_2,\ J_1 > J_2. \tag{27}$$

Now, reformulating (24) as

$$\phi_i = \frac{1}{2} J_{1\cup 2} + \frac{1}{2}\left(J_i - J_j\right), \tag{28}$$

it is easy to see that

$$J_1 > J_2 \implies \phi_1 > \phi_2.$$

For the two-player case, the Shapley value assigns a cost which is a function of the difference of the solitary strategies of the players, centered about one half of the global merger cost.

Now, consider the quantity $\phi_i - J^{(i)}_{1\cup 2}$, i.e., the gap between the *fair* cost defined by the Shapley value for player i, and the cost he actually incurs — the quota $J^{(i)}_{1\cup 2}$ of the merger performance index. Recall, by the efficiency axiom, that $\sum \phi_i = J_{1\cup 2}$. Also, $\sum J^{(i)}_{1\cup 2} = J_{1\cup 2}$. By subtracting these two equalities, we have

$$\phi_1 - J^{(1)}_{1\cup 2} = -\left[\phi_2 - J^{(2)}_{1\cup 2}\right],$$

putting in evidence the fact that there is a player who is excessively benefited — w.r.t. Shapley's distribution of welfare — by its participation in the coalition, and another that experiences the opposite situation. Thus, we can define a unique value for the gap $\epsilon \triangleq |\phi_1 - J^{(1)}_{1\cup 2}|$.

By the transferable utility assumption, we aim to compensate this gap in order to incentivize the formation of a coalition between the players. The cost distribution dictated by (24) can be established by transferring a value $\tau_{\text{TU}} \equiv \epsilon$ from the advantaged player to the other

$$J^{(1)}_{1\cup 2} \pm \tau_{\text{TU}} = \phi_1, \tag{29a}$$

$$J^{(2)}_{1\cup 2} \mp \tau_{\text{TU}} = \phi_2. \tag{29b}$$

To conclude, it is important to recall the situation defined by (21). Notice that, if (19) is strictly satisfied, the allocation computed through (24) always fulfills the following condition:

$$\phi_i < J_i, \quad \forall i \in \{1, 2\} \tag{30}$$

resulting from the individual rationality axiom of the Shapley value. This can be clearly understood by presenting the allocation of costs expressed by ϕ_1 and ϕ_2 from a different perspective. More specifically, we want to show how such allocation relates to the benefit Π yielded by the merger. Taking into account (23), (24) can be rewritten as in (31) in order to explicitly show such relation:

$$\phi_i = J_i - \frac{1}{2}\Pi. \tag{31}$$

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Thus, the Shapley value inherently allocates an equal share of the benefit obtained by the merger to each player, and the final individual payoffs ϕ_i will depend on the costs associated with the solitary strategies. Hence, whenever the surplus produced by merger is positive, individually rational players will always accept to form the merger, provided that utility transfers are allowed.

5.2.4 Bargaining procedure

Following the criteria detailed above, all players *in pairs* will participate — at given time intervals, in general multiple of the sampling time — in a one-shot bargaining, whose outcome will decide the generation of new coalitions. First, at each round, all available pairs of players have to be identified. Notice that the sequence with which the pairs are composed will influence the final outcome of the coalition formation process [45]. Then, each player threatens the other with the unilateral strategy it will take if an agreement is not reached. A threat must not hurt the player who makes it to a greater degree than its opponent (otherwise it would not be credible).

Assumption 6 *Since multiple pairs simultaneously carry on their bargaining, there is no way for a pair of players to be aware of the eventual agreements taking place among the rest of players. Thus, when evaluating the possible formation of coalition $\mathcal{P}_1 \cup \mathcal{P}_2$, each pair assumes that the rest of the agents, i.e., $\mathcal{S} \setminus (\mathcal{P}_1 \cup \mathcal{P}_2)$ remain organized as they were at the previous time step.*

Finally, any pair of players verifies (19) and (22) (where it applies) before stipulating the agreement, following Assumption 6 in the computation of (16).

5.3 Notes on stability

Here we would like to provide some hints regarding the stability of the proposed schemes. One possibility is to employ the framework proposed in [40]. The aim is to ensure that the overall cost of the system is reduced at each time step. However, the autonomous controller structure evolution demands certain assumptions and modifications to the algorithms.

Assumption 7 *We assume that, for each subsystem i considered, there exists a feedback matrix K_i that stabilizes the subsystem. Furthermore, the feedback matrix $\mathbb{K} = \mathrm{diag}(K_i)$ stabilizes the overall system.*

The global feedback matrix $\mathbb{K}$ can be calculated as the solution of an LMI problem, as shown, e.g., in [3]. If a solution to such structured global problem exists, the product of invariant sets for each individual subsystem constitute an invariant set for the overall system. A distributed computation of such invariant sets may be carried out through the method described in [46]. This sets can be used as terminal constraints in the corresponding MPC problem.

Alternatively, following the procedure of [40], whenever an agent or a group of agents plans to deviate from a given default (stable) behavior, the new control sequence is communicated to all the agents affected. These in return communicate their predicted cost variation, that the new input sequence

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produces. If the change is a decrease of the global cost, the proposal is accepted and implemented. In this way, only the control actions that actually improve the behavior of the overall system with respect to a predefined stable behavior are implemented. However, such method of guaranteeing stability requires a forced coordination between agents that do not belong to the same coalition. This may imply that agents in a coalition must keep acting according to their default plan as long as their proposals of deviating from it are not accepted by the affected agents outside the coalition. Moreover, in order to implement such strategy, each agent has to transmit their plans to all neighbors: this means that the system works on the basis of an information broadcast, which breaks the original assumption of autonomous negotiation between pairs of agents. Nevertheless, given that there is no negotiation, this is still less demanding — in terms of communications — than distributed algorithms based on information exchange (such as, e.g., [24]).

5.4 The study of price mechanisms from WP2 for coalition formation[6]

Let $\mathcal{P} = \{\mathcal{P}_i\}$, $i = 1, \dots, |\mathcal{P}|$, denote the set of all coalitions analysed in Subsection 5.2 (e.g., $\mathcal{P}$ is a partition of $\mathcal{S}$). After calculation performed in subsection 5.2 one can create a graph $\mathcal{G}' = (\mathcal{P}, \mathcal{E})$ for which the set of vertices comprises all considered coalitions, while $\mathcal{E}$ contains only those edges between two vertices (coalitions) for which we have computed a positive benefit of the merger, i.e., $(i, j) \in \mathcal{E}$ if we have computed $v(\mathcal{P}_i \cup \mathcal{P}_j)$ and $v(\mathcal{P}_i) + v(\mathcal{P}_j) - v(\mathcal{P}_i \cup \mathcal{P}_j) > 0$. Furthermore, let $\mathcal{W} = \{w_{i,j} \in \mathbb{R} \mid (i, j) \in \mathcal{E}\}$ denote the set of weights for all edges, with $w_{i,j}$ being equal to the value of the benefit of the merger:

$$w_{i,j} := v(\mathcal{P}_i) + v(\mathcal{P}_j) - v(\mathcal{P}_i \cup \mathcal{P}_j) = J_i + J_j - J_{i\cup j}, \quad (i, j) \in \mathcal{E}. \tag{32}$$

A greedy heuristic approach to coalition formation merges only two coalitions $\mathcal{P}_{i^*}$ and $\mathcal{P}_{j^*}$ that achieve the largest benefit $(i^*, j^*) = \arg\max_{(i,j)} w_{i,j}$. However, as pointed out in [47], one can use a more advanced strategy to speed up the merging process by treating all possible mergers simultaneously. We can find this set of combinations by solving a matching problem. For a graph $\mathcal{G}' = (\mathcal{P}, \mathcal{E})$, a subset $\mathcal{M}$ of $\mathcal{E}$ such that no two edges in $\mathcal{M}$ are incident to a common node is called a matching. We allow nodes to remain unmatched and have to solve a maximum weight matching in the graph $\mathcal{G}'$ to obtain the set of combinations producing the greatest benefit

$$\begin{array}{rcll} \mathcal{M}^* & = & \arg\max_{\mathcal{M}} & \sum_{(i,j)\in\mathcal{M}} w_{i,j} \\ & & \text{s.t} & \mathcal{M} \text{ matching } \mathcal{E}. \end{array} \tag{33}$$

The maximum weight matching problem is well studied in the literature, with reported polynomial-time algorithms, for more details see [47, 48] and references therein.

In [49] it is pointed out that the dual of the planar graph for the Minimum Spanning Tree (MST) problem can be interpreted as a bipartite maximum weight perfect matching problem. In that context,

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the parametric analysis of the solution of the MST problem, as carried out in Chapter 6 of DYMASOS deliverable D2.2 ”Report on algorithms for dynamic reconfiguration applied to economics based coordination in SoS”, could be utilized to analyse formation of coalitions.

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6 Distributed coalitions in population control approaches[7]

6.1 Motivating application: demand response management in smart grids

Due to the introduction of distributed and unpredictable energy sources, as for example renewables, into the power grid, the topic of energy management is becoming crucial in power systems. To accommodate for the increased uncertainty in energy supply, it has recently been proposed to match energy demand and supply by regulating the load consumption, together with the energy production. Matching demand and supply can be achieved via the so-called demand response methods, such as *direct control* and *real-time pricing*. While in the former the energy providers have the authority to switch on and off the controlled loads, in the latter the users keep their control authority but are subject to population incentives, such as variable electricity prices, usually proportional to the instantaneous total energy demand. The latter scenario has been recently analyzed using noncooperative game theory and convex optimization [50, 51]. Among other applications, demand response methods have been studied to compute optimal charging strategies for large populations of Plug-in Electrical Vehicles (PEVs) [52, 53].

Following the lines of [54], the effect of demand response methods can be modeled by assuming that each agent i schedules its demand $x^i = \left[x^i_1, \ldots, x^i_T\right] \in \mathbb{R}^T$ over the horizon $\mathcal{T} = \{1, 2, \ldots, T\}$ by solving the following optimization problem

$$x^{i\star}(\mathcal{A}) := \begin{array}{ll} \arg\min\limits_{x\in\mathbb{R}^T} & \theta \left\|x - \hat{x}^i\right\|^2 + (\lambda\mathcal{A} + p_0)^\top x \\ \text{s.t.} & x \in \mathcal{X}^i \end{array} \tag{34}$$

where $\mathcal{A} = \frac{1}{N}\sum_{i=1}^N x^i \in \mathbb{R}^T$ is the vector of the population aggregate demand for the time interval $\mathcal{T}$, and the set $\mathcal{X}^i$ models physical constraints, such as demand bounds and rates, and user preferencies, including intertemporal constraints. The first term in the cost function in (34) models the curtailment cost that each agent encounters for deviating from its nominal energy schedule $\hat{x}^i$, according to the Taguchi loss function [55], where $\theta > 0$ is a constant conversion coefficient. The second term models the price that each agent has to pay for the required energy according to an affine price function $p(\mathcal{A}) := \lambda\mathcal{A} + p_0$, where $\lambda > 0$ is a parameter related to the elastic pricing and p_0 is the basic, possibly time-varying, price for unitary energy consumption. Since the price function depends on the aggregate strategy of all the players, this problem can be addressed via deterministic mean field game formulations [56, 57, 58].

In the literature, the incentive $p(\mathcal{A})$ is typically broadcast to the users by a central operator that computes ahead of time the aggregate consumption of the loads over the whole time horizon $\mathcal{T}$ and updates the incentive accordingly, see [50, Figure 2]. In the setup described above, the individual agents are selfish, in the sense that they do not cooperate. Clearly, it is possible that the agents could benefit from local interactions and cooperations among other (neighbouring) agents, which motivates the analysis of coalitions of agents.

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6.2 Distributed coalitions in population control approaches

In this section, we consider a general scenario where the agents are grouped in coalitions via a communication network, so that they have the chance to cooperate locally. Specifically, we consider a quasi-aggregative game played between the agents of the population, and we assume that the N agents communicate according to a row stochastic matrix $P \in \mathbb{R}^{N \times N}$, whose element $P_{ij} \in [0,1]$ indicates the strength (or relevance) of the communication from agent j to agent i, where $P_{ij} = 0$ denotes no communication from agent j to i, and the diagonal elements are set to zero, that is $P_{ii} = 0$ for all is. The structure of the matrix P determines the *local coalitions* of agents. In the following, we denote by $\mathcal{N}^i$ the set of neighbors of agent i, that is $\mathcal{N}^i := \{j \in \{1, \ldots, N\} \mid P_{ij} > 0\}$. Note that we consider j to be a neighbor of agent i if agent i *receives* communications from j. Moreover, we assume that the interaction between neighbors is not one-to-one, but each agent i is influenced only by the aggregate strategies of its neighbors $\mathcal{N}^i$. More in detail, each agent i tries to minimize a cost function $J^i(x^i, \sigma^i)$ that depends on its own deterministic state $x^i \in \mathcal{X}^i \subseteq \mathbb{R}^n$ and on the ith *coalition state*

$$\sigma^i := \sum_{j \neq i} P_{ij} x^j \in \mathbb{R}^n. \tag{35}$$

Formally, each agent $i \in \{1, 2, \ldots, N\}$ aims at solving the optimization problem

$$x^{i\,\star}(\sigma^i) := \arg\min_{x \in \mathcal{X}^i} J^i(x, \sigma^i). \tag{36}$$

Let us consider the class of aggregative games with heterogeneous convex compact constraints $\mathcal{X}^i$ and quadratic cost

$$J^i(x, \sigma^i) := q_i x^\top Q x + 2 \left(C_i \sigma^i + c_i \right)^\top x, \tag{37}$$

where $q_i > 0$, Q, $C_i \in \mathbb{R}^{n \times n}$, $Q \succ 0$, $c_i \in \mathbb{R}^n$.

Since the agents have interest in optimizing their own cost functions J^i given the aggregate information from the local coalition, the following distributed control algorithm has been proposed in [59].

Initialization: Set $k := 0$. Fix $\lambda \in (0,1)$. Each agent i computes the initial coalition state $\sigma^i_{(0)} := \sum_{j \neq i} P_{ij} x^j_{(0)}$ and sets $z^i_{(0)} := \sigma^i_{(0)}$.

Iteration: Each agent i computes its optimal strategy given the internal state $z^i_{(k)}$:

$$x^{i\,\star}_{(k+1)} := \arg\min_{x \in \mathcal{X}^i} J^i(x, z^i_{(k)});$$

each agent i updates the coalition state: $\sigma^i_{(k+1)} := \sum_{j \neq i} P_{ij} x^{j\,\star}_{(k+1)}$;

and updates its internal state: $z^i_{(k+1)} := \lambda z^i_{(k)} + (1 - \lambda) \sigma^i_{(k+1)}$;

$k \leftarrow k + 1$.

Technical conditions on the cost parameters (q_i, Q, C_i) under which the above algorithm is guaranteed to converge to a Nash equilibrium for the population of agents are established in [59].

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6.3 Application to coalitions of smart buildings

The setting introduced in (34) can be also used to analyse coalitions of smart building populations, specifically for heating ventilation air conditioning (HVAC) systems. As suggested in [54], suppose that each smart building schedules optimally its HVAC usage as follows:

$$\begin{aligned} x^{i\star}(\mathcal{A}) := \quad & \arg\min_{x\in\mathbb{R}^T} \quad \theta\gamma^2 \left\|x - \hat{x}^i\right\|^2 + (\lambda\mathcal{A} + p_0)^\top x \\ & \text{s.t.} \quad x^i \in \left[x^i_{\min}, x^i_{\max}\right] \end{aligned} \tag{38}$$

where $\theta > 0$ is the cost coefficient of the Taguchi loss function, $\gamma > 0$ specifies the thermal characteristic of the HVAC system, $\lambda > 0$ is the elastic price constant and p_0 is the basic price for unit of energy consumption. In [54, Theorems 1, 2] it is shown that if

$$\lambda \leq \frac{2\theta\gamma^2}{N-3} \tag{39}$$

then the distributed control algorithm in [54, Equations 8, 9] can be used to compute Nash equilibrium for the problem in (38). Note that the set of elastic prices λ such that (39) holds shrinks linearly as the population size decreases. On the other hand, Algorithm 2 in [59] guarantees convergence to an almost Nash equilibrium for the more general case of convex constraints $x^i \in \mathcal{X}^i$ instead of box constraints, under the less stringent assumption $\lambda < 2\theta\gamma^2$ which is independent of the population size N, via a communication network P such that $\|P\| \leq 1$.

As a particular case, we next consider a hierarchical communication structure where a total of N buildings are grouped in M coalitions. For simplicity of notation, let us assume that each coalition comprises B buildings. At every communication step, the coalition managers compute the aggregate power demand of their buildings, then communicate among each other using a communication matrix P_M and finally compute the incentive signal for their coalitions. With the convention that buildings controlled by the same manager are grouped together in the extended vector and that the manager is the first agent of the corresponding block, the above coalition management scheme corresponds to a communication matrix

$$P := \underbrace{\left(I_M \otimes \begin{bmatrix} 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix}\right)}_{\text{incentives}} \underbrace{(P_M \otimes I_B)}_{\text{communication}} \underbrace{\left(I_M \otimes \begin{bmatrix} 1/B & \cdots & 1/B \\ 0 & \dots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \dots & 0 \end{bmatrix}\right)}_{\text{coalition aggregation}} = P_M \otimes \frac{1}{B}\mathbb{1}_B\mathbb{1}_B^\top. \tag{40}$$

It is possible to show that if P_M satisfies basic connectivity conditions, then also the matrix P in (40) does, and that $\|P_M\| \leq 1$ implies $\|P\| \leq 1$. Therefore, it follows from [59] that the iterative algorithm proposed above can be used to drive the smart buildings of the coalitions towards a Nash equilibrium solution in a distributed fashion.

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7 Conclusion

As a preliminary result of the research conducted on the coalitional framework foreseen as a part of the DYMASOS project (Task 3.2 of WP3), this document presents two coalitional control architectures specifically developed to address the management of systems of systems. Furthermore, the population-based coalitional scenario developed within Task 1.3 of WP1 is introduced.[8]

One of the critical points that have to be kept into consideration is the *diversity* characterizing the systems in object, yielding very complex interactions between the agents involved (see, e.g., the AYESA and HEP case studies contemplated within the DYMASOS project). In such settings, information about the relevance is of critical importance [1, 2]. Both cooperative and noncooperative game theory frameworks provide a vast knowledge base for this task [3, 4, 5]. However, a critical computational complexity is naturally associated with the analysis and the control of evolving coalition structures emerging by the interaction of a significant (for real-world applications) number of agents [45]. Such complexity will be addressed in further detail over the subsequent Tasks 3.3 and 3.4, where implementation aspects of coalitional algorithms, as well as information-aware mechanisms, will be directly addressed.

Ongoing work involves the inclusion of splitting processes into the algorithms, and the extension of the present study to multiple-player cooperative games. The role of cooperation costs on the outcome of the coalition formation, as well as its relation with control optimality, constitutes an interesting topic for future research.

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References

[1] T. H. Summers and J. Lygeros, "Optimal Sensor and Actuator Placement in Complex Dynamical Networks," in *IFAC World Congress*, Aug. 2014.

[2] Y.-Y. Liu, J.-J. Slotine, and A.-L. Barabasi, "Controllability of complex networks," *Nature*, vol. 473, pp. 167–173, 2011.

[3] J. M. Maestre, D. Muñoz de la Peña, A. Jiménez Losada, E. Algaba, and E. F. Camacho, "A coalitional control scheme with applications to cooperative game theory," *Optimal Control Applications and Methods*, 2013. [Online]. Available: http://dx.doi.org/10.1002/oca.2090

[4] F. J. Muros, E. Algaba, J. M. Maestre, and E. F. Camacho, "The Banzhaf Value as a Design Tool in Coalitional Control," in *2015 Control and Decision Conference (submitted)*, Osaka, Japan, Dec. 2015.

[5] J. Barreiro-Gomez, C. Ocampo-Martinez, J. Maestre, and N. Quijano, "Multi-objective Model-free Control Based on Population Dynamics and Cooperative Games," in *2015 Control and Decision Conference (submitted)*, Osaka, Japan, Dec. 2015.

[6] J. Tsitsiklis, "Problems in decentralized decision making and computation," Ph.D. dissertation, Massachusetts Institute of Technology, 1984.

[7] J. B. Rawlings and B. T. Stewart, "Coordinating multiple optimization-based controllers: New opportunities and challenges," *Journal of Process Control*, vol. 18, no. 9, pp. 839 – 845, 2008.

[8] B. Stewart, J. Rawlings, and S. Wright, "Hierarchical cooperative distributed model predictive control," in *American Control Conference (ACC), 2010*, June 2010, pp. 3963–3968.

[9] D. D. Šiljak, *Decentralized control of complex systems.* Boston: Academic Press Inc., 1991.

[10] A. Zečević and D. D. Šiljak, *Control of complex systems.* Springer US, 2010.

[11] F. Fele, J. M. Maestre, S. M. Hashemy, D. Muñoz de la Peña, and E. F. Camacho, "Coalitional model predictive control of an irrigation canal," *Journal of Process Control*, vol. 24, no. 4, pp. 314 – 325, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0959152414000560

[12] R. Vadigepalli and F. J. Doyle III, "Structural analysis of large-scale systems for distributed state estimation and control applications," *Control Engineering Practice*, vol. 11, no. 8, pp. 895 – 905, 2003.

[13] J. Maestre, P. Van Overloop, M. Hashemy, A. Sadowska, and E. Camacho, "Human in the Loop Model Predictive Control: an Irrigation Canal Case Study," in *Proceedings of the 2014 Control and Decision Conference*, Los Angeles, CA, USA, Dec. 2014.

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement No 611281

[14] R. R. Negenborn, Z. Lukszo, and H. Hellendoorn, Eds., *Intelligent Infrastructures.* Springer, 2010.

[15] F. Malandrino, C. Casetti, C.-F. Chiasserini, and M. Reineri, "A game-theory analysis of charging stations selection by EV drivers," *Performance Evaluation*, vol. 8384, no. 0, pp. 16 – 31, 2015. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0166531614001163

[16] F. Fele, E. G. Debada, J. M. Maestre, M. A. Ridao, and E. F. Camacho, "Coalition analysis methods reported in literature," ETSI, Universidad de Sevilla, Seville, Spain, Tech. Rep., Jun. 2014. [Online]. Available: http://www.dymasos.eu/outcomes/deliverables/

[17] F. Malandrino, C. Casetti, and C.-F. Chiasserini, "A holistic view of its-enhanced charging markets," *Intelligent Transportation Systems, IEEE Transactions on*, vol. PP, no. 99, pp. 1–10, 2014.

[18] G. de O Ramos, J. Rial, and A. Bazzan, "Self-adapting coalition formation among electric vehicles in smart grids," in *Self-Adaptive and Self-Organizing Systems (SASO), 2013 IEEE 7th International Conference on*, Sept 2013, pp. 11–20.

[19] E. F. Camacho and C. Bordons, *Model Predictive Control in the Process Industry. Second Edition.* London, England: Springer-Verlag, 2004.

[20] J. B. Rawlings and D. Q. Mayne, *Model Predictive Control: Theory and Design.* Nob Hill Publishing, 2009.

[21] R. B. Myerson, *Game Theory: Analysis of Conflict.* Cambridge, MA: Harvard University Press, 1997.

[22] T. Sandholm, K. Larson, M. Andersson, O. Shehory, and F. Tohm, "Coalition structure generation with worst case guarantees," *Artificial Intelligence*, vol. 111, no. 12, pp. 209 – 238, 1999. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0004370299000363

[23] R. R. Negenborn, B. De Schutter, and H. Hellendoorn, "Multi-agent model predictive control of transportation networks," in *Proceedings of the 2006 IEEE International Conference on Networking, Sensing and Control (ICNSC 2006)*, Ft. Lauderdale, Florida, April 2006, pp. pp. 296–301.

[24] A. N. Venkat, "Distributed model predictive control: Theory and applications," Ph.D. dissertation, University of Wisconsin-Madison, 2006.

[25] J. M. Maestre, D. Muñoz de la Peña, and E. F. Camacho, "Distributed model predictive control based on a cooperative game," *Optimal Control Applications and Methods*, vol. 32, pp. 153–176, 2011.

[26] M. Farina and R. Scattolini, "Distributed predictive control: A non-cooperative algorithm with neighbor-to-neighbor communication for linear systems," *Automatica*, vol. 48, no. 6, pp. 1088–1096, Jun. 2012.

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement No 611281

[27] A. Alessio and A. Bemporad, “Decentralized model predictive control of constrained linear systems,” in *Proceedings of the 2007 European Control Conference*, 2007, pp. 2813–2818.

[28] L. Magni and R. Scattolini, “Stabilizing decentralized model predictive control of nonlinear systems,” *Automatica*, vol. 42, pp. 1231–1236, 2006.

[29] P. Trodden and A. Richards, “Adaptive cooperation in robust distributed model predictive control,” in *Control Applications, (CCA) Intelligent Control, (ISIC), 2009 IEEE*, July 2009, pp. 896–901.

[30] M. Jilg and O. Stursberg, “Hierarchical Distributed Control for Interconnected Systems,” in *13th IFAC Symposium on Large Scale Complex Systems: Theory and Applications*, Shanghai, China, Jul. 2013, pp. 419–425.

[31] C. Ocampo-Martinez, D. Barcelli, V. Puig, and A. Bemporad, “Hierarchical and decentralised model predictive control of drinking water networks: Application to Barcelona case study,” *Control Theory Applications, IET*, vol. 6, no. 1, pp. 62–71, 2012.

[32] A. Núñez, C. Ocampo-Martínez, B. De Schutter, F. Valencia, J. López, and J. Espinosa, “A multiobjective-based switching topology for hierarchical model predictive control applied to a hydro-power valley,” in *3rd IFAC International Conference on Intelligent Control and Automation Science*, Chengdu, China, Sep. 2013, pp. 529–534.

[33] F. Fele, J. M. Maestre, and E. F. Camacho, “Coalitional control: a bottom-up approach,” in *Submitted to the 2015 American Control Conference*, Chicago, IL, USA, Jul. 2015.

[34] M. Slikker and A. van den Nouweland, *Social and Economics Networks in Cooperative Game Theory*. Kluwer Academic Publishers, 2001.

[35] T. Rahwan, T. Michalak, M. Wooldridge, and N. R. Jennings, “Anytime coalition structure generation in multi-agent systems with positive or negative externalities,” *Artificial Intelligence*, vol. 186, no. 0, pp. 95 – 122, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0004370212000288

[36] J. M. Maestre, D. Muñoz de la Peña, A. Jiménez Losada, E. Algaba, and E. F. Camacho, “An application of cooperative game theory to distributed control,” in *Proceedings of the 18th IFAC World Congress*, 2011.

[37] M. V. Kothare, V. Balakrishnan, and M. Morari, “Robust constrained model predictive control using linear matrix inequalities,” *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.

[38] J. Schuurmans, “Control of water levels in open-channels,” Dissertation, TUDelft, 1997.

[39] R. R. Negenborn, P. J. van Overloop, T. Keviczky, and B. De Schutter, “Distributed model predictive control for irrigation canals,” *Networks and Heterogeneous Media*, vol. 4, no. 2, pp. 359–380, 2009.

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement No 611281

[40] J. M. Maestre, D. Muñoz de la Peña, E. F. Camacho, and T. Alamo, "Distributed model predictive control based on agent negotiation," *Journal of Process Control*, vol. 21, no. 5, 2011.

[41] E. Camponogara and L. de Oliveira, "Distributed optimization for model predictive control of linear-dynamic networks," *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, vol. 39, no. 6, pp. 1331–1338, Nov 2009.

[42] L. Shapley, "A value for n-person games," *Annals of Math. Studies*, vol. 28, pp. 307–317, 1953.

[43] B. T. Stewart, A. N. Venkat, J. B. Rawlings, S. J. Wright, and G. Pannocchia, "Cooperative distributed model predictive control," *Systems and Control Letters*, 2010.

[44] D. Ray, *A game-theoretic perspective on coalition formation.* Oxford University Press, 2007.

[45] M. O. Jackson, *Social and Economic Networks.* Princeton, NJ, USA: Princeton University Press, 2010.

[46] C. Conte, M. Zeilinger, M. Morari, and C. Jones, "Cooperative Distributed Tracking MPC for Constrained Linear Systems: Theory and Synthesis," in *Conference on Decision and Control (CDC)*, Florence, Italy, Dec. 2013.

[47] R. K. Ahuja, T. I. Magnanti, and J. B. Orlin, *Network flows: Theory, algorithms, and applications.* Englewood Cliffs, NJ, USA: Prentice-Hall, 1993.

[48] A. Schrijver, *Combinatorial Optimization, Polyhedra and Efficiency.* Springer-Verlag, 2003, vol. A.

[49] J. C. Williams, "A linear-size zeroone programming model for the minimum spanning tree problem in planar graphs," *Networks*, vol. 39, no. 1, pp. 53–60, 2002. [Online]. Available: http://dx.doi.org/10.1002/net.10010

[50] A.-H. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Trans. on Smart Grid*, vol. 1, no. 3, pp. 320–331, 2010.

[51] H. Chen, Y. Li, R. Louie, and B. Vucetic, "Autonomous demand side management based on energy consumption scheduling and instantaneous load billing: An aggregative game approach," *IEEE Trans. on Smart Grid*, vol. 5, no. 4, pp. 1744–1754, 2014.

[52] Z. Ma, D. Callaway, and I. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," *IEEE Trans. on Control Systems Technology*, vol. 21, no. 1, pp. 67–78, 2013.

[53] F. Parise, M. Colombino, S. Grammatico, and J. Lygeros, "Mean field constrained charging policy for large populations of plug-in electric vehicles," in *Proc. of the IEEE Conference on Decision and Control*, Los Angeles, California, USA, 2014, pp. 5101–5106.

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement No 611281

[54] K. Ma, G. Hu, and C. Spanos, "Distributed energy consumption control via real-time pricing feedback in smart grid," *IEEE Trans. on Control Systems Technology*, vol. 22, no. 5, pp. 1907–1914, 2014.

[55] G. Taguchi and S. Chowdhury, *Quality Engineering Handbook*, Wiley, Ed., 2007.

[56] R. Pesenti and D. Bauso, "Mean field linear quadratic games with set up costs," *Dynamic Games and Applications*, vol. 3, pp. 89–104, 2013.

[57] F. Parise, S. Grammatico, M. Colombino, and J. Lygeros, "On constrained mean field control for large populations of heterogeneous agents: Decentralized convergence to Nash equilibria," in *Proc. of the IEEE European Control Conference*, Linz, Austria, 2015.

[58] S. Grammatico, F. Parise, M. Colombino, and J. Lygeros, "Decentralized convergence to Nash equilibria in constrained mean field control," *IEEE Trans. on Automatic Control* (cond. accepted) [http://arxiv.org/pdf/1410.4421], 2014.

[59] F. Parise, S. Grammatico, and J. Lygeros, "Network aggregative games: Distributed convergence to Nash equilibria," in *Proc. of the IEEE Conference on Decision and Control* (submitted), Osaka, Japan, 2015.

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement No 611281