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Abstract :

Concerning the coalitional control framework constituting WP3 *Coalitional Games in Systems of Systems* of the Dymasos project, the present document addresses the stability issues of SoS controlled by the coalitional schemes (developed over Task 3.2 of WP3), on both classic and coalition-wise sense. Moreover, as part of Task 3.3, it includes a discussion of the role of information exchange between the different agents acting within a SoS.

Keywords :

Coalitional control, game theory, distributed control, hierarchical control, system partitioning, systems-of-systems

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Revision History

The following table describes the main changes done in the document since it was created.

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The DYMASOS Project

The well-being of the citizens in Europe depends on the reliable and efficient functioning of large interconnected systems, such as electric power systems, air traffic control, railway systems, large industrial production plants, etc. Such large systems consist of many interacting components. The sub-systems are usually managed locally and independently, according to different policies and priorities. The dynamic interaction of the locally managed components gives rise to complex behaviour and can lead to large-scale disruptions as e.g. black-outs in the electric grid.

Large interconnected systems with autonomously acting sub-units are called systems of systems. DYMASOS addresses systems of systems where the elements of the overall system are coupled by flows of physical quantities, e.g. electric power, steam or hot water, etc.

Within the project, new methods for the distributed management of large physically connected systems with local management and global coordination will be developed.

The DYMASOS Consortium consists of:

Participant no.	Participant organisation name	Participant short name	Country
1	Technische Universität Dortmund	TUDO	Germany
2	BASF SE	BASF	Germany
3	HEP-Operator distribucijskog sustava d.o.o	HEP	Croatia
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Contents

1 Executive summary

This deliverable presents the work in progress regarding Tasks 3.1 and 3.2 (for which preliminary results were shown in previous deliverables D3.1 and D3.2) and the recent work relative to Task 3.3, within WP3 *Coalitional Games in Systems of Systems*. The present document is organized into three Sections: Sections 3 and 4 concern the stability issues encountered in the coalitional control framework, respectively on the coalition-wise and classical control theoretic aspects. Section 5 discusses the formation of coalitions of controllers from the information requirements point of view. In particular, the objectives of the analysis proposed in Task 3.3 of WP3 include the characterization of the correspondence between broadened system knowledge and performance, especially useful to quantify the sharing of information in economical terms.

2 Introduction

As introduced in previous reports (see deliverables D3.1 and D3.2 [1, 2]), Dymasos WP3 is concerned —as part of the inherent logistical issues and structural constraints characterizing systems of systems (SoS)— with the possibility that certain components of a large-scale heterogeneous system have selfish interests, thus hindering the free sharing of knowledge of local information across the whole system. Indeed, this constitutes an issue when coupling between privacy-concerned subsystems —that likely have incorrect models of the rest of the system— needs to be dealt with, and non-local information is critical for adequate control feedback.

Nonetheless, in spite of the huge effort dedicated to the development of distributed controllers for large-scale systems (see, e.g., [3, 4, 5, 6]), the analysis of the relevance (and possibly of the relative cost) of the information exchange required by distributed control algorithms has received so far little attention. Indeed, in order to allow fundamental properties of centralized control, such as system-wide optimality and stability, the majority of the literature about distributed control overlooked privacy-related issues in order to focus on the overall system performance. However, when dealing with systems with a strong heterogeneous character, selfish interests may not be neglected [2]. As regards the control of networked systems, the focus has been kept on the transmission medium itself, hence on issues related to limited bandwith, data loss, noisy channel [7].

In relation with coalition formation —and in particular with the case when only a subset of the control agents is willing to exchange information about their subsystems— our interest is on the performance bound of the control loop possibly achievable with *partial system information*. On such line of research, studies have been carried out by [8, 9].

Intuitively, the stronger the interaction among different parts of a system, the denser the communication required between the relative control agents. In several cases the variables of a system can be grouped to highlight weakly coupled blocks, often revealing a natural topology, i.e., one arising from the

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physical structure of the system [10, 11, 9, 12]. In general, it is desirable to formulate control laws based on this topology. Nevertheless, from the coalitional control standpoint, it is arguably critical to characterize the improvement provided by a broader knowledge of the system, and promote the formation of coalitions accordingly.

Thanks to modern network technologies, such as wireless networks, smart sensors, and database possibilities offered by cloud computing, a huge amount of diverse information can be shared across a large-scale system in an efficient and flexible way [13]; also, following the wide diffusion of computationally capable mobile devices, the possibilities offered by human-in-the-loop control systems have been enhanced [14]. The advances introduced by these new technologies can be already identified on infrastructure systems, e.g., in modern traffic, water, and electricity networks [4, 15], emphasizing how central is the role that information plays in their efficient management. Besides the smart grid (see [1] and references therein), a clear example is the great interest towards the utilization of Intelligent Transportation Systems (ITS). A consistent research effort is being devoted to this topic, typically involving different kind of game models in order to grasp the complex phenomena derived by the interaction of its heterogeneous user population. See, e.g., the analysis of the problem of choosing the EV charging station [15], the study of the consequences of a coalitional scenario among charging managers [16], or the setting for enabling Vehicle-to-Grid (V2G) operations through coalitions of users [17].

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3 Coalitional stability

Coalition formation is commonly studied in the form of characteristic function games, where a value $v(\mathcal{C})$ is associated to each possible coalition $\mathcal{C} \subseteq \mathcal{N}$, where $\mathcal{N}$ is the set of the agents (see also [2], Section 3). The value of a coalition structure $\mathcal{S}/\mathcal{C} \equiv \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_{|\mathcal{S}/\mathcal{C}|}\}$ is obtained as the sum of the values of all coalitions constituting it, i.e.,

$$V(\mathcal{S}/\mathcal{C}) = \sum_{\mathcal{C} \in \mathcal{S}/\mathcal{C}} v(\mathcal{C}).$$

Hence, any coalition formation process requires the following two preliminary steps: *(i)* generation of the coalition structure and *(ii)* solution of the optimization problem for each coalition [18, 17]. Then, the optimal coalition structure is found as the one characterized by the highest value V^*.[1]

In [2], Sections 5, we presented a greedy algorithm for coalition formation between control agents. It consists in a bottom-up approach, where the structure of the global control law is adapted to the time-variant coupling conditions of the system, promoting the formation of coalitions *as outcome of a pairwise bargaining procedure*. Here comes the third and final step, i.e., the *(iii)* distribution of the value of a given coalition among its components. We present here a method guaranteeing coalition-wise stability, provided that the core of the associated transferable-utility (TU) game is *nonempty*.[2]

3.1 Introduction

We describe here a transfer scheme applicable to the autonomous coalition formation algorithm presented in Section 5 of [2] (see also [19]).[3] In the following, without loss of generality, we consider rational players aiming at maximizing their individual payoff. Thus, player i will choose an allocation p (associated to a given coalition $\mathcal{C}$) over p' (associated to a different coalition $\mathcal{C}'$) if $p_i > p'_i$. In case $p_i = p'_i$, the player will join the larger coalition.

The transfer scheme is associated to a bargaining process among the agents acting in the system, performed in order to reach an agreement on the allocation of the joint benefit resulting from their cooperation. Such bargaining arise from the fact that some (subset of) agents may claim a better individual payoff by not cooperating. Then, for the coalition to be possible, the joint benefit has to be redistributed in order to satisfy such claims. In particular, we consider that all agents external to the claiming subset must support the demanded amount. After such amount is transferred and the claim is satisfied, a new demand may arise by a different subset of agents, giving rise to an iterative process, that may be finite or not. An interesting property is that, under some assumption, such simple process converges to the core of the considered TU game.

[1] Recall that this problem has been demonstrated to be NP-complete [18].
[2] The reader is referred to [1] for a brief survey on the notions of game theory mentioned in the remainder.
[3] A more rigorous formulation of such transfer scheme, along with the convergence demonstration, can be found in [20].

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3.2 TU transfer scheme

We begin by defining a *normalized* TU game, described by a set of players $\mathcal{S} = \{1, \ldots, n\}$ and a *characteristic function*

$$v : 2^{\mathcal{S}} \to \mathbb{R},$$

mapping each coalition $\mathcal{C} \subseteq \mathcal{S}$ to a normalized value satisfying

$$v(\varnothing) = 0, \quad v(\mathcal{S}) = 1, \quad v(\{i\}) = 0, \, \forall i \in \mathcal{S}. \tag{1}$$

In order to split the value of a coalition among its members, we define the set of *imputations* as[4]

$$\mathcal{I} \triangleq \left\{ p = (p_1, \ldots, p_n) \in \mathbb{R}^n | \sum_{i \in \mathcal{S}} p_i = 1 \wedge p_i \geq 0, \, \forall i \in \mathcal{S} \right\}. \tag{2}$$

In words, (2) designates any vector payoff satisfying the *efficiency* and *individual rationality* properties (the reader is referred to [1], Section 3, for further details). Thus, an imputation guarantees to each member at least as much as it would achieve by playing as a singleton coalition.

For any given imputation p over $\mathcal{S}$, the *excess* is defined for each coalition $\mathcal{C} \subseteq \mathcal{S}$ as

$$e(\mathcal{C}, p) = v(\mathcal{C}) - \sum_{i \in \mathcal{C}} p_i, \tag{3}$$

with $e(\varnothing, p) = 0$. In words, the excess represents the aggregate gain of coalition $\mathcal{C}$'s members in case they participate in the agreement yielding the imputation p (e.g., by merging to another coalition $\mathcal{C}' \subseteq \mathcal{S} \setminus \mathcal{C}$, or by splitting into smaller coalitions) w.r.t. the value they get by playing as the coalition $\mathcal{C}$.

The last concept we recall is the *core* of a coalitional TU game:

$$\mathcal{O} = \{p \in \mathcal{I} | e(\mathcal{C}, p) \leq 0, \forall \mathcal{C} \subseteq \mathcal{S}\}, \tag{4}$$

representing the set of imputations (over $\mathcal{S}$) that cannot be improved by any coalition $\mathcal{C} \subset \mathcal{S}$, i.e., any payoff p such that $\sum_{i \in \mathcal{C}} p_i \geq v(\mathcal{C})$. For the purpose of the transfer scheme that will be presented next, we restate (4) in terms of the *demand* of a set of players $\mathcal{C}$ against a generic allocation p.

Definition 1 (Demand) *A demand against* $p = (p_1, \ldots, p_n)$*,* $\sum_{i \in \mathcal{S}} p_i = 1$*, is a pair* $(\mathcal{C}, d)$ *where* $\varnothing \neq \mathcal{C} \subset \mathcal{S}$*, and* $d \triangleq (d_i)_{i \in \mathcal{C}}$ *is a vector satisfying*

$$d_i \geq 0 \tag{5}$$

$$\sum_{i \in \mathcal{C}} d_i = e(\mathcal{C}, p). \tag{6}$$

A demand is *essential* if $\sum_{i \in \mathcal{C}} d_i > 0$. Notice that there cannot be an essential demand for the grand coalition $\mathcal{S}$.

[4] See also [1].

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In order to achieve the convergence of the proposed transfer scheme (see also [20]), we consider, for a given demanding set of agents $\mathcal{C}$, *uniform subdivision* of the demanded amount among the agents, i.e.,

$$d_i = \frac{e(\mathcal{C}, p)}{|\mathcal{C}|}, \tag{7}$$

where the $|\cdot|$ operator designates the cardinality of a set.

A *satisfaction* to a demand $(\mathcal{C}, d)$ against p is an allocation s such that the agents in $\mathcal{C} \subset \mathcal{S}$ get as much as they are able to by playing as a standalone coalition $\mathcal{C}$. The satisfaction of such demand requires the transfer of an equivalent aggregate amount, equally drawn from the rest of agents. More formally, for every agent $i \in \mathcal{S}$,

$$s_i = \begin{cases} p_i + \frac{e(\mathcal{C},p)}{|\mathcal{C}|}, & \text{if } i \in \mathcal{C}, \\ p_i - \frac{e(\mathcal{C},p)}{|\mathcal{S} \setminus \mathcal{C}|}, & \text{if } i \in \mathcal{S} \setminus \mathcal{C}. \end{cases} \tag{8}$$

A transfer scheme is a sequence of allocation proposals p^k, $k \in \mathbb{N}$, such that p^{k+1} is a satisfaction to a demand against p^k. Moreover, if there exists a $\hat{k}$ such that for all $k \geq \hat{k}$ we have $p^k = p^{\hat{k}}$, the transfer scheme is *finite*, i.e., $e(\mathcal{C}^k, p^k) = 0$ for all $k \geq \hat{k}$.

Remark 2 *Notice that allocations produced in any intermediate iteration of the transfer scheme may not satisfy individual rationality. We assume that this is not an issue, as this constitutes as well a base for a new demand. Furthermore, if the game is convex, the overall dissatisfaction is lowered at each iteration and the transfer scheme converges to an imputation in the core in a finite number of steps.*

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4 System stability

This section presents an analysis of the stability properties of the coalitional control algorithms described in [2], Sections 4 and 5.[5] The analysis is based on the stability results available in the literature regarding switched systems, together with the input-to-state (ISS) stability of decentralized systems.

4.1 Introduction

A switched system is a system described by a finite collection of models, here referred to as *configurations* of the system.[6] The switching between different configurations is governed by given criteria (see, e.g., [23]). A generic discrete-time switched linear system can be described by

$$x(k+1) = A^{(i)}x(k) + B^{(i)}u(k),\ i \in \mathcal{I},\ k \in \mathbb{N}, \tag{9}$$

where $\mathcal{I} \triangleq \{1, \ldots, n_c\}$ is the set indexing each possible configuration of the system, $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the global state and input vectors, respectively; $A^{(i)} \in \mathcal{A}$ and $B^{(i)} \in \mathcal{B}$ are the relative transition matrices, belonging to given sets of possible realizations $\mathcal{A}$ and $\mathcal{B}$.

The criteria upon which the switching is performed can be described through a piecewise constant map, $\varsigma : \mathbb{N} \times \mathbb{R}^n \to \mathcal{I}$. So, at each time step k, there will be an active configuration $i = \sigma(k, x(k))$. [7]

In the remainder, we will consider the availability of a set of global linear feedback laws, each one associated to a given configuration of the system. Furthermore, we will assume unconstrained state and inputs.

4.2 Coalitional control as a switched multi-controller system

Consider the switched discrete-time linear system

$$x(k+1) = (A + BK^{(i)})x(k),\ i \in \mathcal{I},\ k \in \mathbb{N}, \tag{10}$$

where x, A, and B are defined as in (9) —except that only one realization of the state and input matrices is considered, so that $\mathcal{A} \equiv A$ and $\mathcal{A} \equiv B$— and $K^{(i)}(A, B) \in \mathcal{K}$ is a global feedback law stabilizing the system, and $\mathcal{K} : A \times B \to \mathbb{R}^{m \times n}$ denotes the family of linear feedback matrices stabilizing system (10). Each feedback law $K^{(i)}$ is associated to a given coalition structure $\mathfrak{P} = \{C_1, \ldots, C_p\}$, partitioning the system state vector into p disjoint subsets

$$x^{(i)} = (x_1^\intercal, x_2^\intercal, \ldots, x_p^\intercal)^\intercal. \tag{11}$$

[5]Further details about the coalitional control algorithms, as well as some insights on their implementation, can be found in [21, 22, 19].

[6]We will see later that each configuration correspond to a given possible coalition structure of the control agents.

[7]Notice that, in general, the active configuration may depend as well on the previous configuration [23]. This occurs indeed in the two coalitional control schemes presented in [2].

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Consider the presence of a set $\mathcal{S} = \{1, \ldots, N\}$ of agents, all in charge of the control of the system. In particular, for any $i \in \mathcal{I}$, each portion of the state vector can only be measured by a given coalition (singleton coalitions are included here). Then, $K^{(i)}$ is obtained as the result of the following problem.

Problem 3 (Coalitional feedback) *Consider the discrete-time switched linear system described by* (10)*. According to the generic partition of the state vector given by* (11)*,*[8] *the feedback matrix can be decomposed as*

$$K^{(i)} = \begin{bmatrix} K_{11}^{(i)} & K_{12}^{(i)} & \cdots & K_{1p}^{(i)} \\ K_{21}^{(i)} & K_{22}^{(i)} & \cdots & K_{2p}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ K_{p1}^{(i)}, & K_{p2}^{(i)}, & \cdots, & K_{pp}^{(i)} \end{bmatrix} \tag{12}$$

Now, given the partial state feedback dictated by (11)*, the problem consists in finding a matrix $K^{(i)}$ such that any $K_{ab}^{(i)} = 0$ if $a \neq b$ (i.e., block-diagonal $K^{(i)}$), and the system* (10) *is stable.*[9]

Notice that Problem 3 may not have a finite number of solutions.

Problem 4 *Consider all the conditions in Problem 3. Define the control performance index associated to the control law $K^{(i)}$ as*

$$J^{(i)} = \sum_{k=0}^{\inf} x(k)^\intercal (Q + (K^{(i)})^\intercal R K^{(i)}) x(k), \tag{13}$$

Then, for a given configuration i, find the matrix $K^{(i)}$ minimizing the difference $(J^{(i)} - J^*)$, where J^* is the optimal control performance associated to the centralized LQR feedback law (i.e., the one obtained with full state feedback).

At this point, it is important to remark that it is not sufficient to independently guarantee the stability of each one of the configurations. Indeed, even if all the configurations are stable, there might exist some switching signals leading the system to evolve over divergent trajectories [23]. On the other hand, an interesting capability provided by switching is that a proper switching sequence between different unstable configurations can drive the system to a stable behaviour; however, this case is not discussed here.

Hence, it is clear that the way the system switches between different configurations, i.e., how agents join or leave coalitions, plays a fundamental role in the system behaviour. Attention has to be paid to the time instants at which agents are allowed to form, leave, or move over coalitions, and to which coalitions —determining some partition of the global state feedback— are produced at any given time. In the remainder, we will try to delineate the necessary conditions that the evolving coalition structure must fulfill in order to ensure the overall system stability.

[8] For simplicity, the input matrix B is assumed here to be diagonal.

[9] Notice that $K^{(i)}$ will correspond to the decentralized (i.e., only local) feedback just for one configuration $i_0 \in \mathcal{I}$.

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4.3 Stability of switched systems

A first approach to the problem is that of seeking for a Lyapunov function that holds for all configuration [23]. In particular, since we are dealing with linear models, we consider a quadratic Lyapunov function $V(x) = x^\intercal P x$, whose existence conditions can be posed in form of linear matrix inequalities (LMI). Therefore, we look for a matrix $P \in \mathbb{R}^{n\times n}$ such that the following constraints are satisfied for all $i \in \mathcal{I}$

$$P = P^\intercal > 0, \tag{14a}$$

$$(A + BK^{(i)})^\intercal P(A + BK^{(i)}) - P \leq 0, \tag{14b}$$

where (14a) requires matrix P to be symmetric and positive definite, and (14b) is a monotone decreasing condition on the value of the Lyapunov function at each pair of subsequent time steps. Efficient algorithms for the computation of (14) are available. On the other hand, it is important to point out that the existence of a Lyapunov function common to all configurations is only sufficient to guarantee stability, constituting a very demanding condition in practice. Less stringent stability requirements are presented in [24].

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5 Information-aware coalition formation

Information plays a fundamental role in the coordination of multiagent systems [25]. Either attained through direct sensing or communication, information is the basis for the local decisions of agents and, as such, decisive to the emergent global behaviour [25]. A fundamental problem in the inherent distributed decision-making framework associated to coalitional control is how the information flow impacts the achievable performance. Such connection is still largely unexplored [26].

Furthermore, as the architecture of multiagent systems becomes more complex, featuring reconfigurable topologies (see, e.g., [22, 21, 27, 28]), misaligned individual interests (e.g., [19, 29, 30, 15, 16], human-in-the-loop control (e.g., [14]), the relationship between possible restrictions on the global availability of information and the system performance is increasingly unclear [25]. The need for locality emerges from the vastness of the mission space, coupled with communication and sensing limitations, that hinder local controllers from having global knowledge [26].

Does providing agents with additional information always lead to improvements in the performance? On the other hand, can the excess of information be detrimental under some given system-wide perspectives?

The authors of [25] address the questions above. As a simple platform for studying the effect of information on multiagent collaboration, a graph-coloring problem is considered in [25]. In such setting, the authors show the impact of the degree of information into the efficiency of the stable coloring profile — a Nash equilibrium— and into its associated convergence rates. Results demonstrate that an increased amount of information is able to improve the efficiency of the Nash equilibria; on the other hand, it degrades the convergence rate of the distributed color adjustment process, where the agents seek to maximize their utility.

For spatially distributed sensing agents, the work of [31] identifies how a measure of locality in the individual control laws can be translated into a bound on the overall achievable efficiency. In particular, the author of [31] addresses the problem of allocating a collection of agents across a mission space,[10] in order to optimize a given submodular global objective. The relationship between the redundancy of information in the control laws implemented by agents and the achievable efficiency of the overall behaviour is then characterized (bounds are given on the efficiency of the stable solutions). When full information regarding the mission space is available to the agents, the efficiency of the resulting stable solutions is guaranteed to lie within 50% of the optimal. However, as the reach of the information becomes more limited, the efficiency of the stable solutions may be as low as $1/N$ of the optimal, where N designates the number of agents. For further details, the reader is referred to [26, 31].

[10]Such problem belongs to the class of networked resource allocation problem.

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5.1 Partial system information

It is natural to expect that the components of a large-scale heterogeneous system show selfish interests, hindering the free sharing of knowledge of local information across the whole system. Indeed, this constitutes an issue when coupling between privacy-concerned subsystems —that likely have incorrect models of the rest of the system— needs to be dealt with, and non-local information is critical for adequate control feedback. Although in some cases local measurements and input sequences are treated as private information, most distributed control schemes developed over the last decade assume public knowledge of the global system model [8]. Concerning the information exchange in the control of networked systems, researchers mainly focused on the transmission medium itself, hence on issues related to limited bandwith, data loss, noisy channel [7]. In relation with coalition formation —and in particular with the case when only a subset of the control agents is willing to exchange information about their subsystems— our interest is on characterizing the performance achievable by a control loop with *partial system information*. On such line of research, recent studies by [8, 9] are available in the literature.

The fundamental objective of coalitional control is to address the controller design problem on a different perspective with respect to the traditional framework, i.e., where the classic closed-loop objective cannot be given a priori. Taking the standpoint clearly expressed by [8], coalitional control aims at addressing problems whose solution begins by accounting for the constraints on the available plant description. When controller designers have to cope with partial models of the plants, one of the question that naturally arise is [8]: *is it possible to relate the achievable closed-loop performance with the amount of available information?* Such issues have been already studied in the field of Economics, employing tools provided by the game theory. The authors of [8] extend a performance metric —the *competitive ratio*— introduced by [32] for the purpose of quantifying the distance from the optimum of the distributed solution of an LP problem when the information locally available is segmented.

Notice that the above question can be reversed. Indeed, instead of assuming that the constraints on information availability define the problem *a priori*, the following question can be addressed: *given some cost objective, what is the minimal partition of the global system model information necessary to guarantee some performance goal?* In order to address this question, the authors of [8] point out that additional metrics need to be formulated, to allow the characterization of the different partition possibilities, and the related *minimality* notion. Again, possible candidate for such task are already available in the game theory literature, related to multi-agent decision making under partial information [8].

The sharing of information in a networked system can be represented through a *knowledge graph*, where each node stands for the model of a given subsystem, and the edges indicate that the information about the models is available to the agents controlling the pointed nodes. *Connectivity* and the *degree distribution* of such graph can be interpreted in quantitative terms [8]. The following definition of knowledge graph is given in [8].

Definition 5 (Knowledge graph) *Let $\Delta : \mathcal{P} \rightarrow \mathbb{R}^{n\times n}$ be a feedback law design method. The* knowledge graph *associated to* Δ *is a directed graph* $\mathcal{G} = (\mathcal{N}, \mathcal{E})$*, with* $\mathcal{N} = \{1, \ldots, n\}$*, and the set of links defined as*

$$\mathcal{E} = \{(i,j) | \exists \mathbf{P} = (A, x) \in \mathcal{P} \text{ such that } [\Delta(\mathbf{P}]_{i,\cdot} = f(A_{j,\cdot})\},$$

where $A_{j,\cdot}$ *denotes the* j*th row of matrix* A*, and* $[\Delta(\mathbf{P}]_{i,\cdot}$ *the* i*th row of the feedback law synthesized through the map* Δ*.*

The authors of [8] show how, for the family of systems corresponding to (22) and objective function (23), the *deadbeat* strategy $\Delta_0 : [\Delta(\mathbf{P}]_{i,\cdot} = -A_{i,\cdot}$ achieves the best competitive ratio among all decentralized (in the sense of the knowledge graph) controllers. In particular, $r_{\mathcal{P}}(\Delta_0) = 2$ for the considered setting. Then, the problem of improving such performance is addressed. The following theorem is stated, providing a necessary condition [8].

Theorem 6 *Consider the map* $\Delta : \mathcal{P} \to \mathbb{R}^{n \times n}$ *such that its competitive ratio* $r_{\mathcal{P}}(\Delta) < 2$*,* $J_{\mathbf{P}}(\Delta(\mathbf{P})) \leq J_{\mathbf{P}}(\Delta_0(\mathbf{P}))$ *and* $\exists \mathbf{P} \in \mathcal{P}$ *for which the strict inequality holds (i.e.,* Δ *dominates the deadbeat strategy). Then the associated* knowledge graph $\mathcal{G}_\Delta = (\mathcal{N}, \mathcal{E})$ *is* weakly connected*, i.e., for any pair of nodes* (i,j) *there exists an undirected path between them.*

5.2 Coalition formation based on PageRank

In this section, the problem of node aggregation in a graph modeling the interactions among the actors of a SoS is addressed [33]. A prior task that needs to be performed for this purpose is the computation of the value of each node, through the employ of given *measures* of relevance. The most employed measures in network (game) theory are formulated on the basis of the inherent structure of the network; thus, they seldom find a straightforward application in the control of dynamic networked systems. Nevertheless, some proposals specifically related with distributed control have been recently formulated. For example, in [21] the relevance of nodes in a distributed control system is expressed as a function of the cost-to-go of the closed loop system under the different possible network topologies.

The present discussion is based on the PageRank index, a variation of the eigenvector centrality measure (see [33] and references therein for further details). The PageRank index essentially relates the value of a given node to the number of other nodes pointing to it, as well as to their relative importance.

In [34], a coalitional control scheme employing the PageRank index for the characterization of the coalition values is presented. Local control agents are allowed to create links among them with the aim of establishing an *aid network*. The PageRank of the aid network is then used as a means to group the agents. Model predictive control (MPC) is implemented inside each coalition in order to calculate the control actions without a central coordinator.

Furthermore, the work of [34] provide tools for the computation of the relevance of links, from the viewpoint of the nodes they connect. This information can be used to determine whether it is advantageous to incur some cost to be able to include their associated information in the control law. It is

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worth to point out that measures to determine the relevance of links from their related nodes are already present in works like [35] and have been proposed in the context of control systems for system partitioning in [36]. Likewise, a modified version of PageRank is proposed in [37] to the problem of finding optimal input nodes of multi-agent systems. However, it is worth to point out that in these works such measures are derived offline and used for analytical purposes. The algorithm proposed in [34] computes measures in a distributed fashion, using them as a prescriptive tool for dynamically grouping local control nodes. Further details are given in the remainder.

5.2.1 Computation of the PageRank

This section contains a brief description of the computation of the PageRank value [33, 34]. Consider a network represented as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \dots, n\}$ is the set of vertices or nodes and $\mathcal{E}$ is the set of edges representing the links among the nodes. In case that a node i has a link pointing to node j then $(i, j) \in \mathcal{E}$. Recall that the PageRank value of a node is an eigenvector centrality measure. In particular, each node i is given a value $p_i^* \in [0, 1]$, with $\sum_{i \in \mathcal{N}} p_i^* = 1$. This value is defined by the sum of the contributions from all the nodes pointing to it, i.e.,

$$p_i^* = \sum_{j \in \mathcal{N}_i^+} \frac{p_j^*}{n_j}$$

where $\mathcal{N}_i^+ := \{j : (j, i) \in \mathcal{E}\}$ is the set of nodes pointing to node i and n_j is the number of outgoing links of node j. Hence, PageRank assigns relevance based on the assumption that a node having links from other important nodes must also be relevant. Let $p^* := [p_i^*]_{i \in \mathcal{N}}$ be the vector of the PageRanks of all nodes. The PageRank vector p^* is the nonnegative unit eigenvector that corresponds to the eigenvalue 1 of A. Then the calculation of the PageRank can be stated in matrix form

$$p^* = Ap^*, \quad p^* \in [0, 1]^n, \quad \sum_{i \in \mathcal{N}} p_i^* = 1 \tag{15}$$

where A is the so-called hyperlink matrix, a variation of the adjacency matrix, given by

$$a_{ij} = \begin{cases} \frac{1}{n_j} & j \in \mathcal{N}_i^+, \\ 0 & \text{otherwise.} \end{cases}$$

We assume that A is a stochastic matrix, i.e., it verifies $\sum_{j \in \mathcal{N}} a_{ij} = 1$ for all $j \in \mathcal{N}$. As it is shown in [38], in order to have a graph of the Internet network that satisfies this property it is necessary to remove all nodes having no links to other nodes. In order to guarantee the uniqueness and existence of p^*, (15) is slightly modified and the PageRank vector p^* is defined as the solution of

$$p^* = Mp^*, \quad p^* \in [0, 1]^n, \quad \sum_{i \in \mathcal{N}} p_i^* = 1, \tag{16}$$

where M is a convex combination of A and $\mathbf{1}^{n \times n}$, i.e., $M := (1 - m)A + \frac{m}{n}\mathbf{1}^{n \times n}$, $m \in (0, 1)$.

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We finish this section with the introduction of a measure of the relevance of the links of the network. We define the *LinkRank* of an edge (i, j) as the sum of the PageRanks that flow between of nodes i and j, i.e.,

$$l_{ij} = \begin{cases} \frac{p_i^*}{n_i} + \frac{p_j^*}{n_j} & \text{if } (i,j),(j,i) \in \mathcal{E}_i, \\ \frac{p_i^*}{n_i} & \text{if } (i,j) \in \mathcal{E}_i, \\ \frac{p_j^*}{n_j} & \text{if } (j,i) \in \mathcal{E}_i, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, those links that connect nodes characterized by a high PageRank value will be regarded as the most important [34].

5.2.2 Node aggregation based on PageRank

In [33], the PageRank index associated to a given network node —that can be the result of the aggregation of two or more agents— is proposed as the *characteristic function* assigning a value to each of the possible coalitions. First, define the set of control agents in the system as $\mathcal{N} = \{1, \ldots, N\}$. Initially, the interaction within the set of agents can be modeled as the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{E}$ is such that $(i, j) \in \mathcal{E}$, with $i, j \in \mathcal{N}$, if and only if the agent i interacts with agent j. Once a given set $\mathcal{C} = \{a_1, a_2, \ldots, a_c\} \subseteq \mathcal{N}$ of agents positively assess the benefit of merging into a coalition, the graph is updated with the description of the new configuration of the network, which will be called $\mathcal{G}'$. In $\mathcal{G}'$, the coalition will be designated by a node that preserves all the original links relative to each player in $\mathcal{C}$.[11]. For convenience the set $\mathcal{R} = \{r_1, r_2, \ldots, r_r\}$ is also defined as the complimentary set $\mathcal{N} \setminus \mathcal{C}$. The value of the coalition is the PageRank associated to the new node in $\mathcal{G}'$.

The aggregation of nodes can be achieved by means of three auxiliary transformation matrices, namely $\mathbf{P}_\mathcal{C}$, $\mathbf{T}_\mathcal{C}$, and $\mathbf{D}_\mathcal{C}$. Matrix $\mathbf{P}_\mathcal{C}$ is a permutation matrix whose purpose is to rearrange the hyperlink matrix so that the players inside $\mathcal{C}$ appear next to each other in the first columns and rows. In particular, the permutation is given by

$$\mathbf{P}_\mathcal{C} = \left[\mathbf{e}_{a_1} \mathbf{e}_{a_2} \ldots \mathbf{e}_{a_c} \mathbf{e}_{r_1} \mathbf{e}_{r_2} \ldots \mathbf{e}_{r_r} \right],$$

where $\mathbf{e}_i$ denotes a column vector of length $|\mathcal{N}|$ with 1 in the i-th position and 0 in every other position. $\mathbf{T}_\mathcal{C}$ is a transformation matrix whose goal is to aggregate into a single node the members in $\mathcal{C}$. It is given by

$$\mathbf{T}_\mathcal{C} = \begin{bmatrix} \mathbf{1}^{|\mathcal{C}|\times 1} & \mathbf{0}^{|\mathcal{C}|\times|\mathcal{R}|} \\ \mathbf{0}^{|\mathcal{R}|\times 1} & \mathbf{I}^{\mathcal{R}\times|\mathcal{R}|} \end{bmatrix}.$$

Finally, $\mathbf{D}_\mathcal{C}$ is a matrix that guarantees that the new hyperlink matrix $\mathbf{A}'$ is also a stochastic matrix. This is done by normalizing the column corresponding to the coalition

$$\mathbf{D}_\mathcal{C} = \begin{bmatrix} \frac{1}{|\mathcal{C}|} & \mathbf{0}^{1\times|\mathcal{R}|} \\ \mathbf{0}^{|\mathcal{R}|\times 1} & \mathbf{I}^{|\mathcal{R}|\times|\mathcal{R}|} \end{bmatrix}.$$

[11] The links previously existing between the members of the new coalition are converted into self-references for the new node.

The hyperlink matrix of $\mathcal{G}'$ can be derived from the original network $\mathcal{G}$ as follows

$$\mathbf{A}' = \mathbf{T}_{\mathcal{C}}^{\mathrm{T}}\mathbf{P}_{\mathcal{C}}^{\mathrm{T}}\mathbf{A}\mathbf{P}_{\mathcal{C}}\mathbf{T}_{\mathcal{C}}\mathbf{D}_{\mathcal{C}}.$$

Consequently, $\mathbf{M}' := (1-m)\mathbf{A}' + m\mathbf{J}'$, with $\mathbf{J}' = \frac{1}{|\mathcal{R}|+1}\mathbf{1}^{|\mathcal{N}'|\times|\mathcal{N}'|}$. As can be seen, $\mathcal{C}$ is mapped into the first vertex of the graph $\mathcal{G}'$.

The value of coalition $\mathcal{C}$ is defined as the PageRank of the associated node in $\mathcal{G}'$, resulting from the aggregation of the relative nodes in $\mathcal{G}$, i.e., the first component of the new PageRank vector $\mathbf{p}'^{*}$. Hence

$$v(\mathcal{C}) = [1\ 0\dots0]\mathbf{p}'^{*} = \mathbf{e}_1^{\mathrm{T}}\mathbf{p}'^{*}.$$

The properties of $\mathbf{M}'$ allow calculating the eigenvector by means of the power method. To this end, let $\mathbf{p}_0' = \frac{1}{|\mathcal{N}'|}\mathbf{1}^{|\mathcal{N}'|\times 1}$. The PageRank vector can then be calculated as the limit of the sequence generated by

$$\mathbf{p}'[k+1] = M'\mathbf{p}'[k] = (1-m)A'\mathbf{p}'[k] + \frac{m}{|\mathcal{R}|+1}\mathbf{1}^{|\mathcal{N}'|\times 1}$$

when $k \to \infty$, which we denote by $\mathbf{p}'^{*}$.Finally, the characteristic function of the PageRank aggregation game becomes

$$v(\mathcal{G}) = \mathbf{e}_1^{\intercal}\mathbf{p}'^{*}, \tag{17}$$

which provides the PageRank that results from the aggregation of the nodes relative to coalitions in $\mathcal{G}$. Note that (17) depends only on the structure of the network $\mathcal{G}$.

5.2.3 The PageRank difference aggregation game

While knowing which players are expected to provide more PageRank when aggregated into a coalition is valuable information, the net effect of the aggregation still needs to be considered. More specifically, it is interesting to know whether the PageRank of the merge is lower or equal to the sum of the individual PageRanks of the players in the coalition. To this end, we define the PageRank difference aggregation game over the same set of players, with the following characteristic function

$$v_{\text{diff}}(\mathcal{C}) = v(\mathcal{C}) - \sum_{i\in\mathcal{C}} v(i),$$

i.e., the characteristic function $v_{\text{diff}}(\mathcal{C})$ measures the gain or loss of PageRank derived from the aggregation. If $v_{\text{diff}}(\mathcal{C}) > 0$ for a certain coalition $\mathcal{C}$ then the aggregation is rational and the players really have an incentive to perform the coalition. The Shapley value [39] of the difference game provides information about the best players to be aggregated from this perspective. Once the Shapley value $\phi_i(\mathcal{N}, \boldsymbol{v})$ has been computed for each node and all possible coalitions, from (17), the difference aggregation game can be obtained as[12]

$$\phi_i(\mathcal{N}, \boldsymbol{v}_{\text{diff}}) = \phi_i(\mathcal{N}, \boldsymbol{v}) - \boldsymbol{v}(i).$$

To overcome the computational complexity of the Shapley value, a numerical approximation, based on the randomized method in [40], is proposed by [33].

[12]This is possible by linearity of the Shapley value.

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5.2.4 Local approach for node aggregation based on PageRank

Information about all the nodes in the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ may not be available. For such cases, the authors of [33] propose an approximation of $v(\mathcal{C})$ (see (17)) associated to a reduced graph $\mathcal{G}_{\text{sub}}(\mathcal{N}', \mathcal{E}') \in \mathcal{G}$, corresponding to a (local) subset of nodes. A simple heuristic —viable if (sufficiently good) knowledge of the links between the nodes in $\mathcal{N} \setminus \mathcal{N}'$ is available— consists in lumping all nodes in $\mathcal{N} \setminus \mathcal{N}'$ into a single node, and deriving the number of self-references of the resulting combined node. If such knowledge is not available, the following method for the derivation of equivalent subnetworks is proposed in [33].

We start by defining the notion of network equivalence employed in [33].

Definition 7 (Graph PageRank equivalence [33]) *Let $\mathcal{G}$ and $\mathcal{G}'$ be two networks containing a sub-network $\mathcal{G}_{\text{sub}}$. They are* equivalent *in a strict PageRank sense for $\mathcal{G}_{\text{sub}}$ if the PageRank of the nodes in $\mathcal{G}_{\text{sub}}$ is the same in both networks. We denote this by $\mathbf{p}^{\mathcal{G}}(\mathcal{G}_{\text{sub}}) = \mathbf{p}^{\mathcal{G}'}(\mathcal{G}_{\text{sub}})$.*

Hence, the goal is to find a network $\mathcal{G}'$ simpler than $\mathcal{G}$, and equivalent to it in PageRank sense. To this end, the nodes in $\mathcal{G}$ are classified in the following three disjoint categories:

- *external nodes*: a node i is external if $i \in \mathcal{G}$ but $i \notin \mathcal{G}_{\text{sub}}$;
- *mid nodes*: a node i is an mid-node if $i \in \mathcal{G}_{\text{sub}}$ but is linked to a node $j \notin \mathcal{G}_{\text{sub}}$;
- *core nodes*: this group includes all the nodes $i \in \mathcal{G}_{\text{sub}}$ that are linked only to nodes in $\mathcal{G}_{\text{sub}}$.

Let $\mathbf{A}$ and $\mathbf{A}'$ be the hyperlink matrices that corresponds respectively to $\mathcal{G}$ and $\mathcal{G}'$. Without loss of generality, we assume that the elements in $\mathbf{A}'$ are arranged in the following order: external nodes (e), mid nodes (m), and core nodes (c). Thus, $\mathbf{A}'$ has the following structure:

$$\mathbf{A}' = \begin{bmatrix} \mathbf{A}'_{ee} & \mathbf{A}'_{em} & \mathbf{A}'_{ec} \\ \mathbf{A}'_{me} & \mathbf{A}'_{mm} & \mathbf{A}'_{mc} \\ \mathbf{A}'_{ce} & \mathbf{A}'_{cm} & \mathbf{A}'_{cc} \end{bmatrix}. \tag{18}$$

The problem of finding an equivalent network translates in computing an hyperlink matrix $\mathbf{A}'$ according to the following theorem:

Theorem 8 *[33] Let $\mathbf{A}'$ be a nonnegative stochastic matrix described by (18). The networks $\mathcal{G}$ and $\mathcal{G}'$*

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are equivalent in a PageRank sense for a subnetwork $\mathcal{G}_{\text{sub}}$ if the following constraints hold:

$$((1-m)\mathbf{A}' + m\mathbf{J}')\,\mathbf{p}'^{\mathcal{G}'}(\mathcal{G}') = \mathbf{p}'^{\mathcal{G}'}(\mathcal{G}') \quad (19a)$$

$$\mathbf{J}'_{cm} = \mathbf{J}_{cm} \quad (19b)$$

$$\mathbf{J}'_{cc} = \mathbf{J}_{cc} \quad (19c)$$

$$\mathbf{A}'_{mm} = \mathbf{A}_{mm} \quad (19d)$$

$$\mathbf{A}'_{mc} = \mathbf{A}_{mc} \quad (19e)$$

$$\mathbf{A}'_{cm} = \mathbf{A}_{cm} \quad (19f)$$

$$\mathbf{A}'_{cc} = \mathbf{A}_{cc} \quad (19g)$$

$$\mathbf{A}'_{ce} = \mathbf{0} \quad (19h)$$

$$\mathbf{A}'_{ec} = \mathbf{0} \quad (19i)$$

where $\mathbf{p}'^{\mathcal{G}'}(\mathcal{G}')$ is the modified PageRank of $\mathcal{G}'$.

Constraints in (19) preserve the structure of $\mathcal{G}_{\text{sub}}$ and impose that all core and mid nodes get the same PageRank value over the equivalent network, i.e., $\mathbf{p}^{\mathcal{G}}(\mathcal{G}_{\text{sub}}) = \mathbf{p}^{\mathcal{G}'}(\mathcal{G}_{\text{sub}})$.

Finally, the heuristic described before can be applied over the (simpler) equivalent network $\mathcal{G}'$ —obtained following Theorem 8— in order to compute an approximation $v'(\mathcal{C})$ of (17).

The demonstration of Theorem 8 is omitted here. The reader is referred to [33] for a detailed discussion of the topic.

5.2.5 A coalitional control scheme based on the PageRank

Consider a dynamical network determined by the interaction of a set of subsystems $i \in \mathcal{N}$, whose behaviour can be modeled as the following discrete-time linear dynamics:

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k) + d_i(k), \\ d_i(k) &= \sum_{j\in\mathcal{N}_i} A_{ij}x_j(k) + \sum_{j\in\mathcal{N}_i} B_{ij}u_j(k), \end{aligned} \quad (20)$$

where $x_i \in \mathbb{R}^{q_i}$ and $u_i \in \mathbb{R}^{r_i}$ with $i = 1, \ldots, n$ are the local states and inputs, and $\mathcal{N}_i = \{j \in \mathcal{N} : A_{ij} \neq 0 \vee B_{ij} \neq 0\}$ is the set of coupled neighbors of node i. The variable d_i is the influence of the neighbors' states and inputs in the update of x_i. We also assume constraints on the state and the input as follows

$$x_i(k) \in \mathcal{X}_i, u_i(k) \in \mathcal{U}_i. \quad (21)$$

Moreover, assume that the interaction between the subsystems can be schematized over the undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. More specifically, the authors of [34] define the *aid graph* $\mathcal{G}_{\text{aid}} = (\mathcal{N}, \mathcal{E}_{\text{aid}})$, where $\mathcal{N}$ is the set of local controllers and $\mathcal{E}_{\text{aid}}$ is the set of edges representing the directed *aid links* that can be established among the nodes. We say that node i requests aid from node j if $(i, j) \in \mathcal{E}_{\text{aid}}$. Likewise, an *aid offer* from node i to node j is translated into a link $(i, j) \in \mathcal{E}_{\text{aid}}$. Notice that in both cases the goal

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is to increase the PageRank value of agent j. The goal of coalitional control is to produce a switching signal for the dynamical regrouping of local controllers. To this end, the PageRank and the *LinkRank* values to calculate the relevance of the nodes and links inside the aid-network.

The key idea of the algorithm proposed by [34] is that local controllers evaluate their performance according to a certain criterion. Possible criteria include, e.g., the distance with respect to the setpoints, the value of the cost function optimized, the measured disturbances.

Here follows a sketch of the algorithm. At time step k:

1. Each local controller i measures its state and evaluates its performance according to a given criterion.
   - In case the performance is low the agent sends an aid request to its neighbors $\mathcal{N}_i$. This means that *aid links* (i, j) are formed for each $j \in \mathcal{N}_i$.
   - In case the performance is good the agent offers support to its set of neighbors $\mathcal{N}_i$. Each node $j \in \mathcal{N}_i$ accepts the proposal if its performance is low. This means that *aid links* (i, j) are formed for each $j \in \mathcal{N}_i$.
2. The PageRank values of the nodes of the *aid network* are calculated by means of a distributed algorithm [41].
3. An average consensus algorithm is executed so that controllers become aware of the average *LinkRank* value.
4. Each node informs its neighbours about its PageRank value so that *LinkRank* values can be calculated.
5. Links with a *LinkRank* value greater than a certain threshold —that in turn is a function of the current LinkRank mean value— are enabled for control purposes. Links are assumed to be bidirectional. The resulting communication topology imposes a partition on the set of local controllers $\mathcal{N}$ into a set of disjoint coalitions.
6. Inside each coalition, local controllers have full communication and implement a distributed control algorithm [42] to calculate the input sequence in a coordinated fashion. Notice that different coalitions do not communicate with each other.

Remark 9 *In [34] it is assumed that distributed algorithms for PageRank and consensus can be used and computed within the sampling time. Nevertheless, the authors state that such assumption can be relaxed, since* LinkRank *values may be computed over a larger sampling period. Furthermore, this computations are performed in parallel with those regarding the synthesis of control actions.*

For further details, the reader is referred to [33, 34].

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5.3 Performance metrics for distributed control

The work of [8] considers the design of distributed linear quadratic (LQR) controllers, assuming that the access to the plant measures is constrained. Then the connection between the closed-loop performance and the amount of exchanged information is characterized. Furthermore, bounds are provided on the minimal information to be exchanged among control agents in order to achieve an improvement over the performance of the best decentralized (communication-less) scheme. In order to do that, the following *performance metrics* are defined for a family of linear distributed plants $\mathcal{P}$ in the form[13]

$$x(k+1) = Ax(k) + u(k),\ x(0) = x. \tag{22}$$

Each given plant in the family $\mathbf{P} \in \mathcal{P}$ is characterized by a given instance of the pair (A, x), and an objective function

$$J_{\mathbf{P}}(K) = \sum_{k=1}^{\inf} x(k)^\intercal x(k) + \sum_{k=0}^{\inf} x(k)^\intercal K^\intercal K x(k), \tag{23}$$

for all $K \in \mathbb{R}^{n\times n}$. For every $\mathbf{P} \in \mathcal{P}$, there exists a controller $K^*(\mathbf{P})$ such that

$$J_{\mathbf{P}}(K^*) \leq J_{\mathbf{P}}(K),\ \forall K \in \mathbb{R}^{n\times n}.$$

Notice that $K^*(\mathbf{P})$ requires, in general, nonlocal model information. Indeed, considering systems in the form (22), it is expected that the best performance is achieved by a control law synthesized on the basis of the full knowledge of matrix A's entries. We consider generic (suboptimal) feedback control laws $K(\mathbf{P}) \in \mathbb{R}^{n\times n}$, whose design requires restricted —not necessarily localized— knowledge, and compare it to the optimal one through the notion defined next.

Definition 10 (Competitive ratio)

$$r_{\mathcal{P}}(K) = \sup_{\mathbf{P}\in\mathcal{P}} \frac{J_{\mathbf{P}}(K)}{J_{\mathbf{P}}(K^*)}. \tag{24}$$

By definition, the competitive ratio of any feedback design is always larger than one; the closer is to one, the better the controller performs in the worst case instance (among all plants included in the family). In other words, $r_{\mathcal{P}}(K)$ close to one indicates that a given control law performs fairly well for all possible instances of the control problem represented by plants in $\mathcal{P}$. Such notion has been introduced in the field of computer science, as a way of assessing an algorithm's ability to generate on-the-fly output, i.e., before they can access a complete description of the inputs [8]. Another measure proposed by [8] is defined next.

Definition 11 (Domination) *A feedback law K is said to* dominate *another law K' if*

$$J_{\mathbf{P}}(K) \leq J_{\mathbf{P}}(K'),\ \forall \mathbf{P} \in \mathcal{P},$$

with strict inequality for at least one plant $\mathbf{P} \in \mathcal{P}$.

[13] We consider here the discrete-time case.

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Remark 12 *The notions defined above can be generalized to the design methods used to synthesize the control laws, i.e., to maps* $\Delta : \mathcal{P} \to \mathbb{R}^{n\times n}$.

6 Conclusion

In spite of the huge effort dedicated to the development of distributed controllers for large-scale systems, the impact of restricted knowledge of the global system by local control agents —requiring costly transfers of information— has received so far little attention. Privacy-related issues —inherent in SoS— have been generally overlooked by researchers, in order to focus on the overall system performance or on specific data transmission problems (e.g., limited bandwith, channel noise, data loss).

When only a subset of the control agents is willing to exchange information about their subsystems, it is interesting to investigate the performance bounds of the overall control loop. These questions have been already addressed in fields such as economics and computer science, but remain an open challenge in the framework of dynamical systems control. From the coalitional control point of view, it is then critical to characterize the improvement provided by a broader knowledge of the system, and promote the formation of coalitions accordingly. Again, the game theory appears as a natural provider of tools for the analysis of such problems, but the question of whether they can be adapted to large-scale real-time optimization still lingers on, due the intrinsic computational complexity associated with such analysis.

Besides addressing the above questions, the reader can find in the present document a discussion of the stability of coalitional control settings from two different point of views: *(i)* analysis of the stability of coalitions, and the associated conditions of benefit transfers among cooperating controllers, and *(ii)* sufficient conditions for the stability of the system in control theoretic sense, i.e., guarantees of reaching the desired operation setpoint.

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