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Abstract :

This deliverable describes some applications of coalitional control algorithms emerged along the guidelines of Tasks 3.3 and 3.4 of WP3 *Coalitional Games in Systems of Systems*.
On one hand, Task 3.3 addresses the study of methods for the derivation of the optimal coalitional structure and the minimal topology for the exchange of feedback information between controllers. These topics can find application in the optimal partitioning of a SoS. On the other hand, Task 3.4 is concerned with the development of ad hoc indices for coalitional control architectures. Such indices can allow the measure of the relevance of given elements of a SoS, such as controller nodes, data links, etc. In connection with Task 3.3, this constitutes a basis for the development of algorithms for the derivation of an optimal coalitional structure. Besides the system partitioning application, indices of node relevance can be employed to constrain/drive the evolution of the global coalitional structure on a predetermined pattern. For example, the preference on the use of certain data links can be formulated and taken into account in the synthesis of the global feedback laws. Furthermore, the use of the Divisia index, a tool widely known in economics for decomposing and analyzing the trend of variables of interest, is considered to study the influence of the different (coalitions of) controllers in a SoS.

Keywords :

Coalitional control, game theory, distributed control, hierarchical control, system partitioning, systems-of-systems

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The DYMASOS Project

The well-being of the citizens in Europe depends on the reliable and efficient functioning of large interconnected systems, such as electric power systems, air traffic control, railway systems, large industrial production plants, etc. Such large systems consist of many interacting components. The sub-systems are usually managed locally and independently, according to different policies and priorities. The dynamic interaction of the locally managed components gives rise to complex behaviour and can lead to large-scale disruptions as e.g. black-outs in the electric grid.

Large interconnected systems with autonomously acting sub-units are called systems of systems. DYMASOS addresses systems of systems where the elements of the overall system are coupled by flows of physical quantities, e.g. electric power, steam or hot water, etc.

Within the project, new methods for the distributed management of large physically connected systems with local management and global coordination will be developed.

The DYMASOS Consortium consists of:

Participant no.	Participant organisation name	Participant short name	Country
1	Technische Universität Dortmund	TUDO	Germany
2	BASF SE	BASF	Germany
3	HEP-Operator distribucijskog sustava d.o.o	HEP	Croatia
4	INEOS Köln GmbH	INEOS	Germany
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Contents

1 Executive summary

This deliverable describes some applications of coalitional control algorithms emerged along the guidelines of Tasks 3.3 and 3.4 of WP3 *Coalitional Games in Systems of Systems*, concerning the development of methods for coalition formation among the agents of a system of systems (SoS).

On one hand, WP3 Task 3.3 addresses the study of methods for the derivation of the optimal coalitional structure and the minimal topology for the exchange of feedback information between controllers. These topics can find application in the optimal partitioning of a SoS, as shown in Section 5, or in the adjustment of the global control law after a fault has been detected in the system. On the other hand, Task 3.4 is concerned with the development of ad hoc indices for coalitional control architectures. Such indices can allow the measure of the relevance of given elements of a SoS, e.g., controller nodes, data links. In conjunction with Task 3.3, this constitutes a basis for the development of algorithms for the derivation of the optimal coalitional structure. Besides the system partitioning application, indices of node relevance can be employed to constrain/drive the evolution of the global coalitional structure on a predetermined pattern. For example, the preference on the use of certain data links can be formulated and taken into account in the synthesis of the global feedback laws. More details on this are given in Section 4. Furthermore, for the evaluation of the impact of the implementation of coalitional control, the indices considered in Tasks 3.3 and 3.4 allow the evaluation of the performance of coalitional strategies, and their comparison with different (distributed) control schemes. The use of the Divisia index, a tool widely known in economics for decomposing and analyzing the trend of variables of interest, is considered in Section 6 to study the influence of the different (coalitions of) controllers in a SoS. Besides the techniques presented in Sections 4 and 5 of this document, the utilization of the Shapley value in a coalitional control framework was discussed in [1, 2]. While the Shapley value can be viewed as a snapshot of the value of the coalitions, the Divisia index can reveal useful clues when the evolution of the system is analyzed over a given time interval. Such different perspective is particularly interesting for coalitional control systems, where the detection of changes in the dynamic coupling plays an essential role.

2 Introduction

An aspect so far rarely contemplated in distributed control problems is the explicit consideration of individual (local) interests of the components of a complex system. Indeed, in order to allow fundamental properties of centralized control, such as system-wide optimality and stability, the majority of the literature about distributed control focuses on the overall system performance [3–6]. However, when dealing with systems with a strong heterogeneous character, selfish interests may not be neglected.

Such feature can be easily identified on infrastructure systems, e.g., in modern traffic, water, and electricity networks [4, 7]. Besides the smart grid (see [8] and references therein), a clear example in this direction is the great interest towards the utilization of Intelligent Transportation Systems (ITS).

A consistent research effort is being devoted to this topic, typically involving different kind of game models in order to grasp the complex phenomena derived by the interaction of its heterogeneous user population. Some examples are the analysis of the problem of choosing the EV charging station [7], the study of the consequences of a coalitional scenario among charging managers [9], or the setting for enabling Vehicle-to-Grid (V2G) operations through coalitions of users [10].

The possible competition between agents may lead to significant intercoupling effects. A coalitional control formulation specifically aimed at the reduction of the undesired effects of the dynamic coupling was presented in Section 7 of [2]. In Section 6 of this document Some considerations on the employ of the Divisia index for the analysis of the time evolution of mutual disturbances in a SoS are given.

In the scenario considered along WP3, promoting the emergence of cooperating clusters among local controllers is reasonable only when an adequate benefit—sufficient to compensate the cost of cooperation—is foreseen. Indeed, permanent communication across the entire system network can be impractical. Consequently, even when the whole system is owned and managed by a single entity, the use of a traditional centralized control approach is hampered. The decentralization of the control law over several controllers requires the partition of the global system model into a collection of submodels. In general, the dynamics of the resulting submodels will be interdependent. Nevertheless, the strength of the coupling between any pair of subsystems can be sufficiently low to allow the design of a globally stabilizing control law, whose structure is shaped accordingly to the neglected couplings. As a given information flow emerges, a specific partition of the set of controllers into connected components is produced, such that the state feedback information is locally confined within each component [11]. As a result, the system will be governed by a structured (decentralized) global feedback law.

The aforementioned applications emphasize how central is the role that information plays in the efficient management of SoS. However, in the majority of the studies regarding the control of networked systems the focus has been kept on issues related with the transmission medium itself, e.g., limited bandwidth, data loss, noisy channel [12]. The analysis of the relevance (and possibly of the cost) of the information exchange required by distributed control algorithms has received so far little attention.

Knowledge of the relative relevance of the elements of a SoS can be employed to efficiently direct the system's resources where most needed [13, 14]. Cooperative game theory currently represents a basic tool to derive ad hoc metrics [11, 15]. The method for system partitioning presented in Section 5 is a demonstration of the employ of the Shapley value for the measure of the contribution of each available data link in the optimality of the final global feedback.

This kind of analysis must not be necessarily relegated to offline design. Indeed, thanks to the now widely available network technologies, such as wireless networks, smart sensors, and database possibilities offered by cloud computing, a huge amount of diverse information can be shared across a large-scale system in an efficient and flexible way [16]. All these factors contribute to the emergence of a new approach to distributed control problems, where the cooperation between networked controllers is actively fostered and adapted in real-time to the state of the system [17]. Section 4 illustrates how constraints can

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be formulated on the use of the data network infrastructure used for the exchange of feedback information between local controllers. In this way, the evolution of the coalition structure can be driven towards the priority formation of some connected components over others, while having it reflected in the synthesis of the global control law.

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3 Preliminaries

3.1 System description

Let us consider a SoS whose components, each one governed by a local controller, are identified within the set $\mathcal{N}$. The behavior of any subsystem $i \in \mathcal{N}$ follows the linear discrete-time model

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + D_i w_i(k), \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{q_i}$ are respectively the local state and input vectors, constrained in the sets $\mathcal{X}_i$ and $\mathcal{U}_i$ respectively. The vector $w_i \in \mathbb{R}^{m_i}$ represents the measurable disturbances resulting from the coupling with other subsystems,

$$w_i(k) = \sum_{j\in\mathcal{M}_i} A_{ij}x_j(k) + B_{ij}u_j(k) \triangleq \sum_{j\in\mathcal{M}_i} d_{ij}(k), \tag{2}$$

where $\mathcal{M}_i \triangleq \{j \in \mathcal{N} \setminus \{i\} | A_{ij} \neq \mathbf{0} \vee B_{ij} \neq \mathbf{0}\}$ is referred to as the *neighborhood* of subsystem i.

Denoting the global state as $x \equiv \{x_i\}_{i\in\mathcal{N}} \in \mathbb{R}^n$ and the global input as $u \equiv \{u_i\}_{i\in\mathcal{N}} \in \mathbb{R}^q$, the state evolution of the whole SoS is governed by the following equation

$$x(k+1) = Ax(k) + Bu(k), \tag{3}$$

where $A = \{A_{ij}\}_{i,j\in\mathcal{N}} \in \mathbb{R}^{n\times n}$ and $B = \{B_{ij}\}_{i,j\in\mathcal{N}} \in \mathbb{R}^{n\times q}$.

Analogous models have been employed for the control of real large-scale systems such as drinking water networks composed by interconnected water tanks [18], irrigation canals, modeled as integrator-delay cascades in [17, 19, 20], supply chains [21], traffic networks and power grids [22].

3.2 Exchange of information

The set of controllers can communicate through a network infrastructure that can be schematized by the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of enabled links. The description provided by $\mathcal{G}$ delineates a partition $\mathcal{P}(\mathcal{N}, \mathcal{G}) = \{\mathcal{C}_1, \dots, \mathcal{C}_{N_c}\}$ of the set of controllers into N_c connected components, referred to as coalitions [23]. Formally, coalitions are defined as [24]

$$\mathcal{C}_i \subseteq \mathcal{N},\ \mathcal{C}_i \cap \mathcal{C}_j = \varnothing,\ \forall i, j \in \{1, \dots, N_c\}, i \neq j, \text{and} \bigcup_{i=1}^{N_c} \mathcal{C}_i = \mathcal{N}.$$

The number of coalitions N_c pertains to the interval $[1, |\mathcal{N}|]$, whose extremes correspond to the centralized control case (all the $|\mathcal{N}|$ subsystems connected[1]) and the case where each subsystem "forms a coalition" on its own (all links disabled).[2] For the sake of readability, let us define the set $\mathcal{S}_{\mathcal{P}}(\mathcal{N}, \mathcal{G}) = \{1, \dots, N_c\}$ indexing the coalitions characterizing the current partition of the system. The dynamics (1) of all subsystems relative to a given connected component $i \in \mathcal{S}_{\mathcal{P}}$ can be aggregated as

$$\xi_i(k+1) = \mathbb{A}_{ii}\xi_i(k) + \mathbb{B}_{ii}\nu_i(k) + \mathbb{D}_i\omega_i(k), \tag{4}$$

[1]Notice that this condition does not necessarily require all the links to be active.
[2]The $|\cdot|$ operator stands for the cardinality of a set.

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with $\xi_i = \{x_j\}_{j \in \mathcal{C}_i}$ the aggregate state vector, and $\mathbb{A}_{ii} = [A_{jl}]_{j,l \in \mathcal{C}_i}$ the relative state transition matrix, describing the state coupling between members of the same coalition. The vector ν_i and the matrix $\mathbb{B}_{ii}$ are derived by an analogous definition. Finally, the vector

$$\omega_i = \{w_j\}_{j \in \mathcal{C}_i} \tag{5a}$$

gathers the disturbances due to the coupling with subsystems external to $\mathcal{C}_i$. Following (2) it holds that

$$w_j = \sum_r d_{jr}(k), \text{with } r \in \mathcal{M}_j \setminus \mathcal{C}_i,\ j \in \mathcal{C}_i \tag{5b}$$

pointing out how, for each $j \in \mathcal{C}_i$, the set of unknown coupling from neighboring subsystems is reduced to the neighbors left out of the coalition. That is, from the coalition standpoint, the uncertainty comes from any subsystems $r \in (\bigcup_{j \in \mathcal{C}_i} \mathcal{M}_j) \setminus \mathcal{C}_i$. The composition of $\mathbb{D}_i$ follows accordingly.[3]

[3]In case of singleton coalition, i.e., $\mathcal{C} \equiv \{i\}$, the description given by (4) coincides with (1).

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4 Constrained coalitional structure evolution

In [11,17] the evolution of the coalitional structure follows a supervisory management based on a trade-off between optimal performance and reduction of communication requirements of the control law (decentralization). In particular, a specific feedback law is associated with each communication topology; this linear control law is structured accordingly, so as to circumscribe the interchange of state information within the coalition. Given the current state x of the system, the topology $\mathcal{E}$ to be implemented is chosen over a candidate set (assume here the complete set of possible topologies $\mathcal{N} \times \mathcal{N}$) as the one minimizing the following cost function:

$$\min_{\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}} J(x, \mathcal{E}) = x^\intercal P(\mathcal{E}) x + \rho c |\mathcal{E}|, \tag{6}$$

where $c \in \mathbb{R}_+$ is the cost of the use of one link, and $\rho \in \mathbb{R}_+$ a parameter balancing the upper bound on the closed-loop performance and the cost of communication that a given topology $\mathcal{E}$ implies.

Let's analyze more in detail how $\mathcal{E}$ and its associated control law $K(\mathcal{E})$ are connected in the formulation of [11]. Since the joint optimization of the manipulated variables and of the network topology constitutes a complex mixed integer problem,[4] an approximated formulation of the problem is proposed in [11] with the objective of decoupling such variables in the optimization. In particular, the selection of the data network topology is made independently of the future control inputs by evaluating an upper bound of the predicted system performance. Thus, the fundamental problem addressed is the following.

Problem 1 *Given the system* (3) *controlled by a structured linear feedback law* $u = K(\mathcal{E})x$*—where* $K(\mathcal{E}) = \mathrm{diag}\{K_1, K_2, \ldots, K_{N_c}\}$*—find a Lyapunov function* $V(x) = x^\intercal P(\mathcal{E}) x$ *for the closed-loop system such that it provides an upper bound of the infinite horizon cost of the system under the given structured control law.*

If such Lyapunov function exists, the following condition is fulfilled as the closed-loop state trajectory evolves:

$$x(k+1)^\intercal P x(k+1) + x(k)^\intercal Q x(k) + u(k)^\intercal R u(k) \leq x(k)^\intercal P x(k), \tag{7}$$

expressing the decrease of the infinite-horizon cost of the trajectory left until the equilibrium is reached. Condition (7) can be posed in matrix inequality form as [25]

$$P(\mathcal{E}) > 0, \tag{8a}$$

$$(A + BK(\mathcal{E}))^\intercal P(\mathcal{E})(A + BK(\mathcal{E})) - P(\mathcal{E}) \leq -Q - K(\mathcal{E})^\intercal R K(\mathcal{E}). \tag{8b}$$

Furthermore, the exchange of state information allowed by the given network topology $\mathcal{E}$ is reflected in the structure of the associated feedback matrix through the following constraints:

$$\{i, j\} \notin \mathcal{E} \Rightarrow K^{ij} = K^{ji} = \mathbf{0}, \tag{9}$$

for all $i, j \in \mathcal{N}$, $i \neq j$.

[4] See Section 3.5 in [2].

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At this point, several pairs $\{K(\mathcal{E}), P(\mathcal{E})\}$ can be admissible as a solution to Problem 1. The search can be restricted by looking for the $P(\mathcal{E})$ describing the ellipsoid $x^\intercal P(\mathcal{E})x = 1$ with greatest possible volume. The volume of such ellipsoid is related to the determinant of $P(\mathcal{E})$: minimizing $\det(P(\mathcal{E}))$ is tantamount to maximizing the volume of the ellipsoid [25]. However, this results in a nonconvex optimization. An alternative convex formulation can be obtained by minimizing the geometric mean of the eigenvalues [26]. Hence, the following cost function is considered

$$\min_{P(\mathcal{E})} \det(P(\mathcal{E}))^{1/n}. \tag{10}$$

At this point, Problem 1 translates to

Problem 2 *For each topology $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$, find the pair $\{K(\mathcal{E}), P(\mathcal{E})\}$ satisfying* (10)*, subject to* (8) *and* (9)*.*

Notice that Problem 2 is subject to matrix inequality constraints. Direct products between the decision variables $K(\mathcal{E})$ and $P(\mathcal{E})$ in (8) prevent its solution via semidefinite programming (SDP). Nevertheless, by proper transformations, the problem can be rendered in linear matrix inequalities (LMI) form. This is done by means of the Schur complement and by applying the substitutions $W(\mathcal{E}) = P(\mathcal{E})^{-1}$ and $Y(\mathcal{E}) = K(\mathcal{E})P(\mathcal{E})^{-1}$. Then, it is possible to rewrite (8) as

$$\begin{bmatrix} W & WA^\intercal + Y^\intercal B^\intercal & WQ^{1/2} & Y^\intercal R^{1/2} \\ AW + BY & W & \mathbf{0} & \mathbf{0} \\ Q^{1/2}W & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ R^{1/2}Y & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} > 0, \tag{11}$$

where $\mathbf{I}$ and $\mathbf{0}$ are identity and zeros matrices of proper dimensions (the dependence on $\mathcal{E}$ of $W(\mathcal{E})$ and $Y(\mathcal{E})$ has been dropped in the notation for clarity). Structural constraints (9) can be applied to the new decision variables

$$\{i, j\} \notin \mathcal{E} \Rightarrow Y^{ij} = Y^{ji} = \mathbf{0}. \tag{12}$$

Remark 1 *Although a linear feedback formulation has been assumed throughout this section, the same concepts can be applied to a receding-horizon approach, as shown in [17] (see also [2]). In that case, constraints on the structure of $P(\mathcal{E})$ should be applied similarly to* (9)*.*

In the formulation presented so far, the data network influences the control law through constraint (9). The evolution of the coalition structure during the system operation follows the minimization of (6). Next we present a method to condition the evolution of the coalition structure, setting a *preference* on the use of some links and, at the same time, reflecting it in the synthesis of the associated control laws.

4.1 Feedback design with constraints on the links' value

This section, based on the work of [27], presents a method to set a preference on the choice of the data network topology, in the form of constraints on the Shapley value associated to the data link game. To

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this aim, the definition of the data link game is first provided. Then, the constraints imposed to the LMI problem for the synthesis of the structured feedback laws are described.

4.1.1 Data link cooperative game

The correspondence between any network topology $\mathcal{E}$ and the performance of the associated system partition can be evaluated by formulating a cooperative game whose players are the data links between the controllers. On the basis of such formulation, the relevance of any link $l \in \mathcal{N} \times \mathcal{N}$ is measured in [27] by its resulting Shapley value: this knowledge can then be used to extract the most cost-efficient system partition.

Let us now define the characteristic function $v(\mathcal{E}, x) : 2^{N_l} \mapsto \mathbb{R}$ for the data link cooperative game ($N_l = |\mathcal{N} \times \mathcal{N}| = \frac{1}{2}|\mathcal{N}|(|\mathcal{N}| - 1)$ denotes the number of links). This function associates a real value to each of the 2^{N_l} possible configurations of the set $\mathcal{N} \times \mathcal{N}$ of links. In particular, the value of any subset $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ of activated links is defined as the composition of a performance index and a term representing the cost incurred for the use of the network infrastructure:

$$v(\mathcal{E}, x) = x^\intercal P x + \rho c|\mathcal{E}|. \tag{13}$$

Performance index The first term in (13) is an upper bound over the cost-to-go of the system governed by the feedback law $K(\mathcal{E})$ starting from the state x:

$$x(k)^\intercal P(\mathcal{E}) x(k) \geq J(k), \tag{14}$$

where $P(\mathcal{E})$ is a positive-definite matrix providing a convex Lyapunov function for the closed-loop system, that can be obtained by solving Problem 2. Then the following condition holds:

$$x(k+1)^\intercal P x(k+1) + x(k)^\intercal Q x(k) + u(k)^\intercal R u(k) \leq x(k)^\intercal P x(k), \tag{15}$$

expressing the decrease of the cost-to-go as the closed-loop state trajectory evolves following the optimal control law.

Cost of communication It is expected that the effort required for the coordination is proportional to the number of agents involved in a coalition, involving non-negligible data exchange. Therefore, costs required for the cooperation of a given set of agents can be taken into account by means of ad-hoc indices related to such communication requirements. The second term in (13), that is $\rho c|\mathcal{E}|$, expresses the cost of enabling the data links characterizing $\mathcal{E}$. Note that although a single constant price c is considered here for the use of each link, the method admits the use of differentiated prices as well. The link cost-of-use c is extended to the infinite horizon by the factor ρ.

We assume such communication costs comparable with the control performance index, such that the stage cost incurred by the entire system at time k will be

$$\ell(k) = x(k)^\intercal Q x(k) + u(k)^\intercal R u(k) + c|\mathcal{E}|. \tag{16}$$

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A value for each possible collection of enabled links has been obtained so far. Next, a measure of the relevance of each single link is derived through the Shapley value.

4.1.2 Relevance of data links

By viewing a network topology as a *coalition of links*, the contribution of each link $l \in \mathcal{E}$ to the cost $v(\mathcal{E}, x)$ can be derived employing solution concepts such as the Shapley value:

$$\phi_l(v) = \sum_{\mathcal{E} \subseteq (\mathcal{N} \times \mathcal{N}) \setminus \{l\}} \frac{|\mathcal{E}|!(N_l - |\mathcal{E}| - 1)!}{N_l!} \left[v(\mathcal{E} \cup \{l\}) - v(\mathcal{E}) \right], \tag{17}$$

where $|\mathcal{E}|$ is the number of enabled links characterizing the topology $\mathcal{E}$. The value of $\phi_l(v)$ gives an insight on the relevance of the data link $l \in \mathcal{N} \times \mathcal{N}$ on the performance of the system in closed-loop with all feedback laws where that link is employed. In particular, the lower the value, the more critical is the link contribution towards the optimal performance of the system—since (13) represents a cost to be minimized.

By formulating specific constraints on the synthesis of the feedback laws, the evolution of the coalition structure can be driven to the preferential use of some links. Two classes of constraints are contemplated in [27]: *(i) absolute*, where the Shapley value of a given link is limited under/over a given threshold, i.e.,

$$\begin{aligned} \phi_l(v) &> \underline{\phi}_l, \\ \phi_l(v) &< \overline{\phi}_l, \end{aligned} \tag{18}$$

and *(ii) relative*, when the constraint on the Shapley value of a given link is expressed with respect to another link's value:

$$\phi_{l_a}(v) > \phi_{l_b}(v). \tag{19}$$

Furthermore, consider possible state constraints in the ellipsoidal form

$$\mathcal{X} = \{x | (x - x_c)^\intercal G (x - x_c) \leq \varsigma\}, \tag{20}$$

where $G \in \mathbb{R}^{n \times n}$ is a positive definite matrix, $x_c \in \mathbb{R}^n$ is a constant vector describing the center of the ellipsoid, and $\varsigma > 0$.

Before getting to the actual constraint formulation, notice that (17) can be computed in matrix form as:

$$\boldsymbol{\phi}(v) = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N_l} \end{bmatrix} = \mathbf{M} \begin{bmatrix} v(\mathcal{E}_1, x) \\ v(\mathcal{E}_2, x) \\ \vdots \\ v(\mathcal{E}_{N_\mathcal{E}}, x) \end{bmatrix} = \mathbf{M}\mathbf{v}, \tag{21}$$

where $N_\mathcal{E} = 2^{N_l}$, $v(\mathcal{E}_i, x)$ computed according to (13), and any element m_{ij} of $\mathbf{M} \in \mathbb{R}^{N_l \times 2^{N_l}}$ is defined as [28]

$$m_{ij} = \begin{cases} \frac{(|\mathcal{E}|-1)!(N_l - |\mathcal{E}|)!}{N_l!} & l \in \mathcal{E}, \\ -\frac{|\mathcal{E}|!(N_l - |\mathcal{E}|-1)!}{N_l!} & l \notin \mathcal{E}. \end{cases} \tag{22}$$

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Notice that, for any given $i \in \{1, \dots, N_l\}$,

$$\sum_{j \in \mathcal{N}_{\mathcal{E}}} m_{ij} c |\mathcal{E}| \equiv c,$$

where $\mathcal{N}_{\mathcal{E}} = \{1, \dots, 2^{N_l}\}$ is the set of the indices designating each of the possible network topologies, such that $j \mapsto \mathcal{E}_j \subseteq \mathcal{N} \times \mathcal{N}$. Then, according to (21), (17) can be rewritten as

$$\phi_i(v) = c + \sum_{j \in \mathcal{N}_{\mathcal{E}}} m_{ij}(x^\intercal P(\mathcal{E}_j)x),\ i \in \{1, \dots, N_l\}. \tag{23}$$

Now, using a standard S-procedure [25], constraints (18) can be reformulated in order to include state constraints (20) as

$$\phi_l(v) + \gamma(\varsigma - x^\intercal G x) < \overline{\phi},$$
$$\phi_l(v) - \gamma(\varsigma - x^\intercal G x) > \underline{\phi},$$

where $\gamma \in \mathbb{R}$ and, without loss of generality, $x_c = 0$. Then, using (23), (18) becomes

$$\begin{aligned} \sum_{j \in \mathcal{N}_{\mathcal{E}}} m_{ij}(x^\intercal P(\mathcal{E}_j)x) + \gamma(\varsigma - x^\intercal G x) < \overline{\phi} - c, \\ \sum_{j \in \mathcal{N}_{\mathcal{E}}} m_{ij}(x^\intercal P(\mathcal{E}_j)x) - \gamma(\varsigma - x^\intercal G x) > \underline{\phi} - c, \end{aligned} \tag{24}$$

Constraints (19) are reformulated as

$$\sum_{j \in \mathcal{N}_{\mathcal{E}}} m_{rj}(x^\intercal P(\mathcal{E}_j)x) > \sum_{j \in \mathcal{N}_{\mathcal{E}}} m_{sj}(x^\intercal P(\mathcal{E}_j)x), \tag{25}$$

where $r, s \in \{1, \dots, N_l\}$, $r \neq s$. Notice that (25) does not include constraints on the state, because it would not result in an LMI formulation. For the sake of brevity, constraints in this section are not expressed in their final LMI form. For details on the implementation on a SDP solver, the reader is referred to [27].

4.2 Computation algorithm

The computation of the feedback laws fulfilling the constraints on the Shapley value is carried out over two steps. In the first step, the pair $\{K(\mathcal{E}), P(\mathcal{E})\}$ is obtained by the solution, for each possible $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$, of

$$\max_{Y(\mathcal{E}), W(\mathcal{E})} \det(W(\mathcal{E}))^{1/n}, \tag{26}$$

subject to (11) and (12). Once the pair $\{Y(\mathcal{E}), W(\mathcal{E})\}$ is computed, the values for $K(\mathcal{E})$ and $P(\mathcal{E})$ are obtained through the inverse substitution $P(\mathcal{E}) = W(\mathcal{E})^{-1}$ and $K(\mathcal{E}) = Y(\mathcal{E})W(\mathcal{E})^{-1}$.

In the second step, the constraints on the Shapley value are addressed. Notice that these constraints involve $P(\mathcal{E})$. Thus, its value must be recomputed by solving the following problem

$$\min_{\gamma, \mathcal{P}} \sum_{j \in \mathcal{N}_{\mathcal{E}}} \det(P(\mathcal{E}_j))^{1/n} - \gamma, \tag{27}$$

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where $\mathcal{P} \triangleq \{P(\mathcal{E}_1), P(\mathcal{E}_2), \ldots, P(\mathcal{E}_{N_\mathcal{E}})\}$, $N_\mathcal{E} = 2^{N_l}$, i.e., the set of all matrices P associated to each one of the possible network topologies. Indeed, while Step 1 is carried out independently over each topology, the optimization in Step 2 concerns the whole set of topologies at once. The optimization in (27) is subject to (8) (for all $P \in \mathcal{P}$) by considering the values of $K(\mathcal{E})$ computed in Step 1, and Shapley value constraints (24) and (25).

4.3 Suboptimality index

The satisfaction of the additional constraints on the value of the links in the global feedback laws produces some degradation of the performance. Hence, it is interesting to measure such degradation through a suboptimality index $\eta \in (0, 1]$ defined as

$$\eta = \frac{x^\intercal P(\mathcal{E}^*)x}{x^\intercal \bar{P}(\mathcal{E}^*)x}, \tag{28}$$

where $\mathcal{E}^*$ is the topology that, given x, minimizes (6), and $\bar{P}(\mathcal{E}^*)$ is the *ideal* value of $P(\mathcal{E}^*)$, defined as

$$\bar{P}(\mathcal{E}^*) = \lim_{n\to\infty} \sum_{t=0}^{n} (A_{\mathrm{CL}}^\intercal)^t (Q + K(\mathcal{E}^*)^\intercal R K(\mathcal{E}^*)) A_{\mathrm{CL}}^t, \tag{29}$$

where $A_{\mathrm{CL}} = A + BK(\mathcal{E}^*)$.

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5 Partitioning as link game

The decentralization of the control law over several controllers requires the partition of the global system model into a collection of submodels. The problem of deriving an optimal system partition can be addressed by considering the analogous problem of the formation of *static* coalitions. Notice that this constitutes an NP-complete problem, since it requires the evaluation of all the possible partitions of the set $\mathcal{N}$, whose number grows exponentially with the number of agents (for instance, a set of 10 agents yields $115,975$ possible partitions). Such exponential trend is described by the Bell number [29]. Let $\mathcal{K}_s$ denote the set of possible coalitional structures (that is, partitions of $\mathcal{N}$) composed by s coalitions. The cardinality of this set is expressed by the Stirling number of the second kind [30]:

$$|\mathcal{K}_s| = \frac{1}{s!}\sum_{j=0}^{s-1}(-1)^j\binom{s}{j}(s-j)^{|\mathcal{N}|}. \tag{30}$$

The Bell number evaluates (30) over all possible sizes of a partition:[5]

$$|\mathcal{K}| = \sum_{s=1}^{|\mathcal{N}|}|\mathcal{K}_s|. \tag{31}$$

In general, the dynamics of the submodels resulting from the partition will be interdependent. Nevertheless, the strength of the coupling between any pair of subsystems can be sufficiently low to allow the design of a globally stabilizing control law, whose structure is shaped accordingly to the neglected couplings. Details about the synthesis of such control law are provided next.

5.1 Control law

Consider the system setting described in Section 3. The system is composed by a collection of interconnected controller nodes. The topology of the resulting network is described by the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. The set of enabled links $\mathcal{E} \subseteq \mathcal{N}\times\mathcal{N}$ produces a partition $\mathcal{P}(\mathcal{N},\mathcal{G}) = \{\mathcal{C}_1,\ldots,\mathcal{C}_{N_c}\}$ of the set of controllers into connected components, such that the state feedback information is locally confined within each component. As a result, the system will be governed by a structured (decentralized) global feedback law:

$$u(k) = K(\mathcal{E})x(k), \tag{32}$$

where $K(\mathcal{E}) = \mathrm{diag}\{K_1, K_2, \ldots, K_{N_c}\} \in \mathbb{R}^{q\times n}$ is the block diagonal linear feedback matrix—and $K_i \in \mathbb{R}^{q_i\times n_i}$ is the submatrix associated to the state information shared between members of coalition $\mathcal{C}_i$. In other words, the state trajectory of any coalition $\mathcal{C}_i \in \mathcal{P}(\mathcal{N},\mathcal{G})$ will evolve according to

$$\xi_i(k+1) = (\mathbb{A}_{ii} + \mathbb{B}_{ii}K_i)\xi_i(k) + \mathbb{D}_i\omega_i(k), \tag{33}$$

where ξ_i denotes the state vector of $\mathcal{C}_i$, and ω_i is the disturbance due to subsystems external to $\mathcal{C}_i$, whose coupling is neglected in the representation given by $\mathcal{P}(\mathcal{N},\mathcal{G})$.[6] We assume here that the objective of the

[5] $|\mathcal{C}| \in [1, |\mathcal{N}|]$, see Section 3.2.

[6] See (4) in Section 3.2.

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distributed control scheme is to drive the system's state toward the origin of the state space. To this aim, a global performance index—measuring the distance of the state trajectory from the origin as well as the control effort—is minimized.

$$J(k) = \sum_{t=k}^{\infty} x(t|k)^{\intercal} Q x(t|k) + u(t|k)^{\intercal} R u(t|k) = \sum_{t=k}^{\infty} x(t|k)^{\intercal}(Q + K(\mathcal{E})^{\intercal} R K(\mathcal{E})) x(t|k), \tag{34}$$

where $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{q\times q}$ are positive (semi-)definite weighting matrices.

Remark 2 *The number of possible topologies that can be derived on a fully connected network of* $\mathcal{N}$ *nodes is* $N_l = |\mathcal{N} \times \mathcal{N}| = \frac{1}{2}|\mathcal{N}|(|\mathcal{N}| - 1)$*. Notice that this value is greater than the total number of distinct partitions described by* (31)*. Nevertheless, by considering the links as players, the problem can be addressed through a standard cooperative game model, allowing to obtain a solution in closed form thanks to the Shapley value formulation.*

5.2 Shapley value for system partitioning

The definition of the data link game given in Section 4.1 is summarized next. Then, the constraints imposed to the LMI problem for the synthesis of the structured feedback laws, as presented in [31], will be described.

Consider a game $v(\mathcal{E}, x) : 2^{N_l} \mapsto \mathbb{R}$ that, given the value of the state x, associates a real value to each of the 2^{N_l} possible configurations of the set $\mathcal{N} \times \mathcal{N}$ of links. This is done by considering the upper bound given by a Lyapunov function for the system, together with the cost associated to the use of a given structured feedback law and its underlying network topology:

$$v(\mathcal{E}, x) = x^{\intercal} P x + \rho c|\mathcal{E}|, \tag{35}$$

where $P(\mathcal{E})$ is a positive-definite matrix describing a convex Lyapunov function for the closed-loop system, providing an upper bound for the global system performance

$$x(k)^{\intercal} P(\mathcal{E}) x(k) \geq J(k).$$

The pairs $\{K(\mathcal{E}), P(\mathcal{E})\}$ associated to each network topology can be derived from the solution of an LMI problem, where the structure of the feedback law is imposed as an additional constraint. For details about its formulation the reader is referred to Section 4 of this document.

The second term evaluates the cost of enabling the data links characterizing $\mathcal{E}$, where $c \geq 0$ is the cost-of-use associated to each link, and $\rho > 0$ is a parameter used to extend this cost over the infinite-length horizon (so as to make the two terms in (35) comparable). The characteristic function defined as in (35) allows to derive a partition of the global system such that the best tradeoff is achieved between the performance (34) and the associated cost of coordination between local controllers. A further step is required, namely the measure of the relevance of the data links.

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Given the data link cooperative game $v(\mathcal{E}, x)$, the relevance of any single link $l \in \mathcal{N} \times \mathcal{N}$ expressed by the Shapley value [32] is

$$\phi_l(v) = \sum_{\mathcal{E} \subseteq (\mathcal{N} \times \mathcal{N}) \setminus \{l\}} \frac{|\mathcal{E}|!(N_l - |\mathcal{E}| - 1)!}{N_l!} \left[v(\mathcal{E} \cup \{l\}) - v(\mathcal{E}) \right]. \tag{36}$$

This value represents the expected average contribution to the cost reduction in the control of the global system. Due to the way (35) is defined, the lower the value the better.

An aspect that still needs to be discussed is the dependence of the characteristic function (and thus of the Shapley value) on the state of the system. The objective is to evaluate the possible partitions over the expected range of operation of the system, in order to be able to assess the relevance of links independently of the current state. To this aim, a statistical approach is employed in [31].

Assume the state of the system belongs to the set $\mathcal{X} \subset \mathbb{R}^n$. As a consequence, the values taken by (35) and (36) will be confined to closed intervals

$$\begin{aligned} v(\mathcal{E}, x) &\in [v_{\min}, v_{\max}],\ x \in \mathcal{X}, \mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}, \\ \phi_l(v) &\in [\phi_{l,\min}, \phi_{l,\max}]. \end{aligned} \tag{37}$$

Then, the probability density of the Shapley value for each link can be constructed by evaluating (13) on a sufficiently large number of samples over the state space. Consider a subdivision $\mathcal{H}_l = \{h_1, h_2, \ldots, h_{N_h}\}$ of the interval $[\phi_{l,\min}, \phi_{l,\max}]$, such that the interval $h_i = [\bar{h}_i - \delta, \bar{h}_i + \delta)$. Furthermore, $h_1 - \delta = \phi_{l,\min}$ and $h_{N_h} + \delta = \phi_{l,\max}$. A set of counters $\mathcal{R}_l = \{r_1, \ldots, r_{N_h}\}$ is associated to the intervals, such that any h_i has an associated counter r_i. The following procedure is proposed in [31]:

1. Extract a random sample $x \in \mathcal{X}$.
2. Compute the value of the characteristic function (35) for each of the 2^{N_l} possible topologies, and then derive the Shapley value for each link through (36).
3. For each link $l \in \mathcal{N} \times \mathcal{N}$, if $\phi_l(v) \in h_i$, then $r_i = r_i + 1$.

The above steps are repeated until the desired sampling density has been reached.

Once the values of the counters r_i are normalized by $N_s/(2\delta)$, where N_s is the number of samples considered, the mapping $\mathcal{H}_l \mapsto \mathcal{R}_l$ constitutes a discretized probability density $g_l(\phi_l)$ of a normal distribution, whose mean can be calculated as

$$\mu_l = \int_{\phi_{l,\min}}^{\phi_{l,\max}} \phi_l g(\phi_l) d\phi_l. \tag{38}$$

The value provided by (38) can be interpreted as a measure of relevance in the performance of the closed-loop system. According to this value, the links can be finally arranged in decreasing order of relevance, and the system partition can be performed evaluating the best trade-off between performance and communication cost. For further details, the reader is referred to [31].

5.3 Note for a receding-horizon implementation

Consider the case in which a receding-horizon scheme is used, and (34) is restated on a finite horizon by means of a final stage cost. The performance index to be minimized takes the form

$$J(k) = \sum_{t=0}^{N_p-1} x(t|k)^\intercal Q x(t|k) + u(t|k)^\intercal R u(t|k) + x(N_p|k)^\intercal P(\mathcal{E}) x(N_p|k). \tag{39}$$

Then, in order to keep the assumption of decentralized state feedback information, $P(\mathcal{E})$ should be a block-diagonal positive-definite matrix $P(\mathcal{E}) = \text{diag}\{P_1, P_2, \ldots, P_{N_c}\} \in \mathbb{R}^{n\times n}$, whose blocks are associated to the coalitions $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{N_c}\}$ produced by the network topology $\mathcal{E}$ (see also Section 3.2).

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6 Application of economic criteria to coalitional control

The logarithmic mean Divisia index (LMDI) is commonly used in economics to analyze the evolution of certain state indicators, taking as components of such variation some related variables [33]. As part of Task 3.4 of WP3, the interest here is on the development of indices that can be used as key performance indicators for the assessment of control strategies and, furthermore, as a way to determine the evolution of the topology of the control architecture. In the remainder, a method based on the use of the LMDI for decomposing and analyzing the influence of the different (coalitions of) controllers in a SoS is presented. We have shown in previous works (see [1, 2] and references therein, as well Sections 4 and 5 in this document) some examples of the utilization of the Shapley value in a coalitional control framework. Indeed, while the Shapley value provides a snapshot of the value of the coalitions, the LMDI can reveal useful clues when the time evolution of the system is considered. Such different perspective is particularly interesting for coalitional control systems, where the detection of changes in the dynamic coupling plays an essential role. The utilization of the Divisia index in the coalitional control framework is object of undergoing study. Some preliminary considerations are provided in the remainder.

6.1 Introduction

In most distributed model predictive control (MPC) algorithms available in the literature [3, 34, 35] the cooperation structure of local controllers is static, which implies permanent communication between coupled controllers. However, while communication is needed for coordination, it may not always be strictly necessary. As the set of controllers jointly steer the subsystems to the desired setpoint, the state of the coupling between these subsystems changes until it can be eventually neglected. At this point, it may be preferable to switch off the communication. On these grounds, the methods proposed along DYMASOS WP3 are characterized by dynamic grouping of local controllers into cooperating sets. We refer to such architecture as *coalitional control*.

The problem of estimating the impact of individual local controllers and coalitions from an economic point of view is addressed here. The proposed estimation is based on the identification of the responsible actors for changes occurring in certain aggregate indicators, through the use of the LMDI index. Widely known in economics, the Divisia index constitutes a tool for describing the changes in a variable over time as a function of given components.

6.2 LMDI for coalitional control

Consider the setting described in Section 3, i.e., a system resulting from the composition of several subsystems (identified in the set $\mathcal{N}$), responding to the linear discrete-time model (1). The objective of the control is to regulate the state of the system to the origin. The closed loop performance of each

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subsystem at each time step is measured through the quadratic stage cost

$$\ell_i(k) = x_i(k)^\intercal Q_i x_i(k) + u_i(k)^\intercal R_i u_i(k), \tag{40}$$

where $Q_i \in \mathbb{R}^{n_i \times n_i}$ and $R_i \in \mathbb{R}^{q_i \times q_i}$ are positive (semi-)definite weighting matrices.

The aggregation of local controllers is allowed with the objective of achieving superior performance, by reducing the undesired effects of the dynamic coupling between subsystems. Notice that this is reasonable only when an adequate benefit is foreseen, sufficient to compensate the cost of cooperation.

Therefore, the goal is to determine how the mutual coupling affects the performance of the SoS from both global and local perspectives, paying attention to other factors such as the structure of the system and the weight of the coalition regarding the overall performance. To this end, we focus on the disturbances generated by the local controllers, as defined in (2). Behind the evolution of the coalitional structure that characterizes coalitional control lies an evaluation—possibly over a future time window—of the outcome of coalitions to be formed. On these grounds we consider the Divisia index as possible candidate for such task. We proceed by first defining the variable

$$\varpi_i(k) = \sum_{t=0}^{N_p-1} \|w_i(t|k)\|, \tag{41}$$

as the estimate of the disturbance affecting subsystem $i \in \mathcal{N}$ during the time interval $[k, k+N_p-1]$ ($N_p \geq 1$). Let us define as well the estimated performance of the system along the same time horizon as

$$\tilde{J}(k) = \sum_{t=0}^{N_p-1} x(t|k)^\intercal Q x(t|k) + u(t|k)^\intercal R u(t|k), \tag{42}$$

$$\tilde{J}_i(k) = \sum_{t=0}^{N_p-1} x_i(t|k)^\intercal Q_i x_i(t|k) + u_i(t|k)^\intercal R_i u_i(t|k), \tag{43}$$

where $Q = \text{diag}\{Q_1, \dots, Q_{|\mathcal{N}|}\}$ and $R = \text{diag}\{R_1, \dots, R_{|\mathcal{N}|}\}$ are positive (semi-)definite global weighting matrices.

At this point, we propose the following decomposition of the aggregate perturbation affecting the system during a given time interval:

$$\Pi = \sum_{i\in\mathcal{N}} \|\varpi_i\| = \sum_{i\in\mathcal{N}} \tilde{J}\frac{\tilde{J}_i}{\tilde{J}}\frac{\|\varpi_i\|}{\tilde{J}_i} = \tilde{J}\sum_{i\in\mathcal{N}} S_i I_i, \tag{44}$$

where $S_i \triangleq \frac{\tilde{J}_i}{\tilde{J}}$ and $I_i \triangleq \frac{\|\varpi_i\|}{\tilde{J}_i}$. Following the decomposition in (44), we consider that the increment in the overall level of disturbances along the time interval $[k, k+N_p-1]$ can be ascribed to three factors:

$$\Delta\Pi = \Pi(k) - \Pi(k_0) = \Delta\Pi_{\mathbf{P}} + \Delta\Pi_{\mathbf{S}} + \Delta\Pi_{\mathbf{I}}, \tag{45}$$

where k_0 is the reference time instant. According to $\Delta\Pi_{\mathbf{P}}$, the change in the overall disturbance is connected to the variation in the system performance, measured as

$$\Delta\Pi_{\mathbf{P}} = \sum_{i\in\mathcal{N}} \alpha_i \log\frac{\tilde{J}(k)}{\tilde{J}(k_0)}. \tag{46}$$

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The relation with the change in the balance between local and global costs is captured by

$$\Delta\Pi_{\mathbf{S}} = \sum_{i\in\mathcal{N}} \alpha_i \log \frac{\tilde{S}_i(k)}{\tilde{S}_i(k_0)}, \tag{47}$$

and finally

$$\Delta\Pi_{\mathbf{I}} = \sum_{i\in\mathcal{N}} \alpha_i \log \frac{\tilde{I}_i(k)}{\tilde{I}_i(k_0)} \tag{48}$$

captures the variation of the intensity of the disturbance affecting subsystem i w.r.t. its local performance index (43). The weights α_i are defined as

$$\alpha_i = \frac{\|\varpi_i(k)\| - \|\varpi_i(k_0)\|}{\log\|\varpi_i(k)\| - \log\|\varpi_i(k_0)\|}. \tag{49}$$

The proposed method can provide a different perspective on the evaluation of the performance of a SoS: this is particularly interesting for coalitional control systems, where the detection of changes in the coupling plays an essential role. An application of such indices as a criterion to drive the evolution of the topology of the control architecture is foreseen. Moreover, these indices can find their way as key performance indicators for the assessment of distributed control strategies in general.

7 Conclusion

It is clear how central is the role that information plays in the efficient management of SoS. Over Tasks 3.3 and 3.4 information-related aspects of coalitional algorithms have been addressed. Indeed, from the coalitional control point of view, it is critical to characterize the improvement provided by a broader knowledge of the system, and promote the formation of coalitions accordingly. Of course, this comes at a cost of a more intense information exchange. Nevertheless, the knowledge of the relative relevance of the elements of a SoS can be employed to efficiently allocate the system's resources where most needed. Cooperative game theory currently represents a fundamental tool to derive ad hoc metrics. The method for system partitioning presented here is a demonstration of the employ of the Shapley value for the measure of the contribution of each available data link in the optimality of the final global feedback law.

The availability of a measure of the relevance of the information flows opens the way to the formulation of constraints on the use of the data network infrastructure used for the exchange of feedback information between local controllers. As illustrated in this document, the evolution of the coalition structure can be driven towards the priority formation of some connected components over others, while having it reflected in the synthesis of the global control law.

An aspect so far rarely contemplated in distributed control problems is the explicit consideration of individual (local) interests of the components of a complex system. The possible competition between agents may lead to significant intercoupling effects. The last section of this document is dedicated to some considerations on the employ of the Divisia index for the analysis of the time evolution of mutual disturbances in a SoS.

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